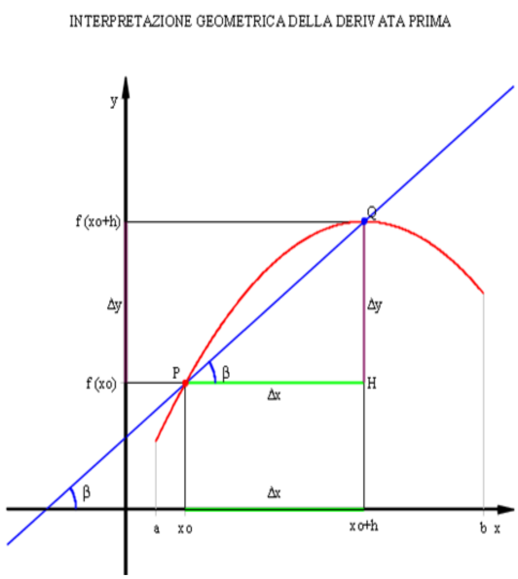
Derivatives

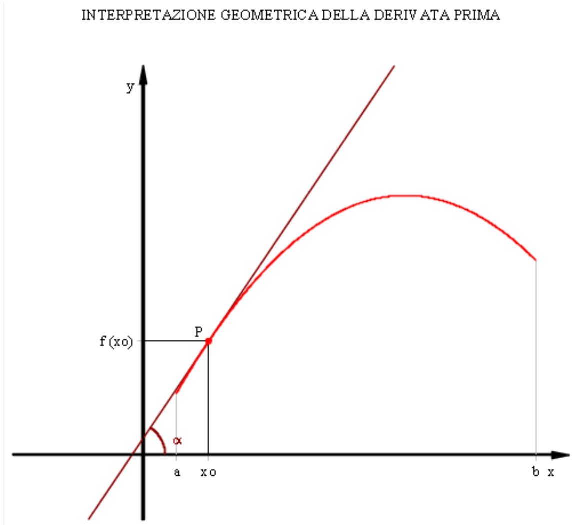
Obiettivi:

1) Conoscere le derivate
2) Saper applicare tale strumento alla risoluzione dei problemi della realtà
3) Migliorare le competenze di "Analisi "
4) Migliorare le competenze della lingua inglese

Welcome to the presentation on derivatives.

What is this? Any ideas on this topic?





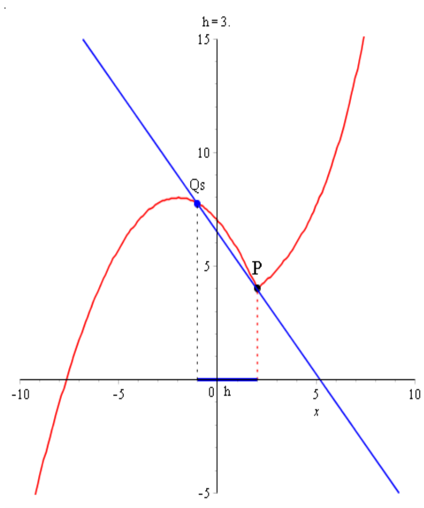
We have two graphs. In these graphs there are straight and curve lines.

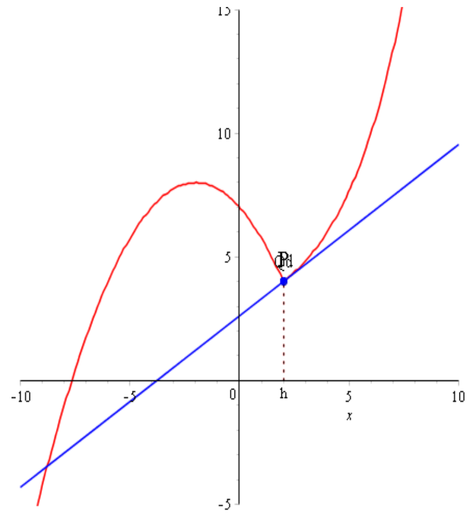
- Which is the difference between the two graphs?
- What is the difference between the two straight lines?

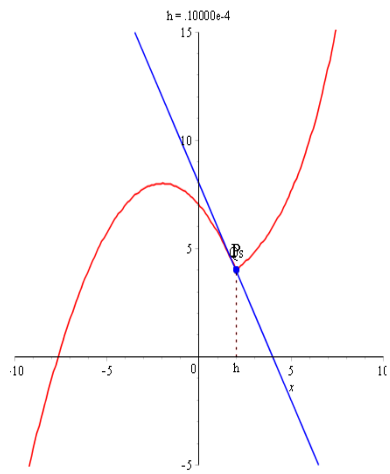
Answer

The first is secant and the second is tangent to the same curve .
The tangent is the limit position of the secant.

Now we see three graphs.







What is the difference in these graphs?
What is the position of the line compared to the curved line?
What is the difference between the first, second and third straight line in these graphs?

Answer

In the first graph the line is secant, in the second and third graphs the lines are tangents to the curve at the same point; but we see two types of tangents, the slopes of the two tangents exist but are different.
In the second graph Q approaches P from the right while in the third graph Q approaches P from the left

The two tangents are different. Point P is called the *corner point.*

Derivatives and their properties

The focus of this Unit will be the study of derivatives and their properties. I think that by studying this subject, Maths starts to become a lot more fun than it was just a few topics ago.

Well let's get started with our derivatives.

I know it sounds very complicated, but the derivative rules are very simple. You don't need to think/reflect.

What is a derivative?
It is a very important mathematical tool.

Where did you find this concept?

1) In the definition of the slope of the tangent to a curve at a point

2) In instant velocity.

3) Whenever we have a difference quotient $\frac{\Delta y}{\Delta x}$ in which Δx approaching 0

By the way, can you give me a definition of "instant velocity"?
It is a change in ………………… and ………………… at a particular point in …………………

The changes …………………at a particular point in a graph.

It is a change in direction and speed at a particular point in time

The changes occur at a particular point in a graph.

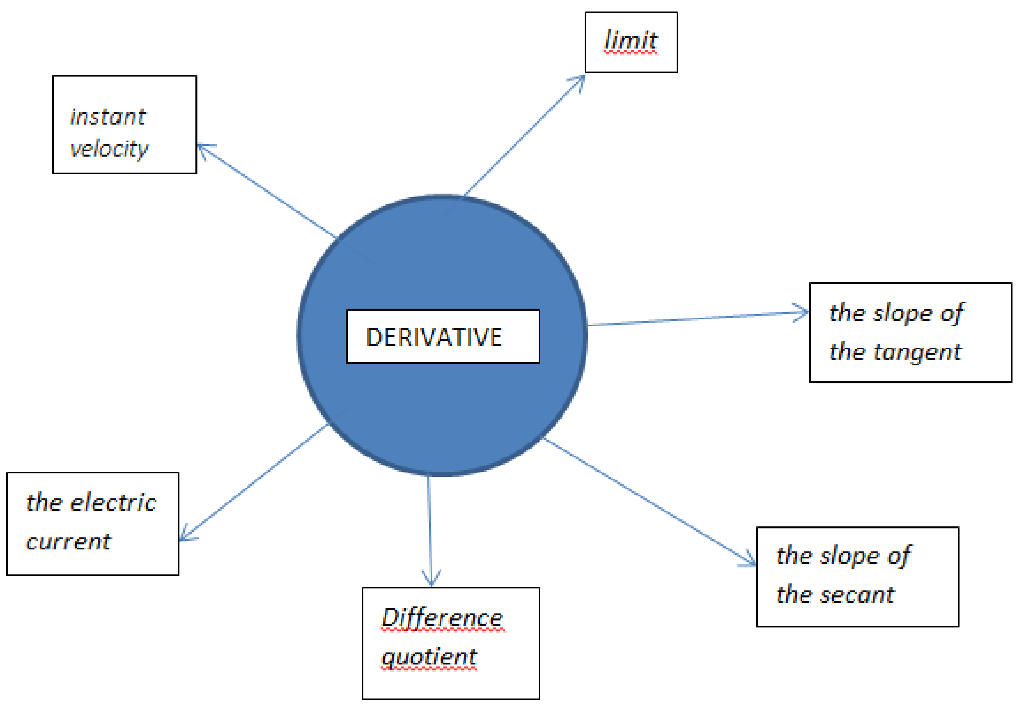
That's not enough. Can you give me a more precise definition?

It is the limit of .... which tends ....approaching........

It is the limit of mean velocity when Δt is approaching zero

What is the meaning of the words we find in the following MIND MAP?

MIND MAP



Here is a link to a youtube video on Derivatives:
https://www.youtube.com/watch?v=rAof9Ld5sOg

(Draw on the whiteboard what described in the following passage)

Well, that’s my coordinate axes.
In general, if I have a straight line, and I ask you to find the slope,
I think you already know how to do this: it’s just the change in y divided by the change in x.

We know that slope is the same across the whole line, but if I want to find the slope at any point in this line,
what I would do is I would pick a point x – say, I’d pick this point.

I could pick any two points, the choice it’s pretty arbitrary, and I would figure out what the change in y is.
This is the change in y, delta y, that’s just another way of saying change in y, and this the change in x, delta x.

We figured out that the slope is defined really as change in y divided by change in x.
Another way of saying that is delta y divided by delta x,very straightforward.

Now, what happens, though, if we’re not dealing with a straight line?

Another coordinate axes.

Let’s just say I had the curve y equals x squared .
Let me draw it in a different colour.
So y equals x squared looks something like this.
It’s a curve, you’re probably pretty familiar with it by now.

Now what I’m going to ask you is: what is the slope of this curve?

What does it mean to take the slope of a curve now?
Well, in this line the slope was the same throughout the whole line.
But if you look at this curve, the slope changes, right?
Here it’s almost flat, and it gets steeper steeper steeper steeper steeper until gets pretty steep.

So, you’re probably saying, well, how do you figure out the slope of a curve whose slope keeps changing?

Well there is no slope for the entire curve. For a line, there is a slope for the entire line, because the slope never changes.

But what we could try to do is figure out what the slope is at a given point. And the slope at a given point would be the same as the slope of a tangent line.

How are we going to figure out what the slope is at any point along the curve y equals x squared?

That's where the derivative comes into use, and now for the first time you'll actually see why a limit is actually a useful concept.

CONTEXTUALIZATION

Problem 1 - A car is travelling along a straight road. The space path varies according to a law of the motion given by a certain function s (t). What is its speed, i.e., how does the space covered vary, instant by instant, with respect to the time taken to travel?

Problem 2 - The height of a missile in metres, t seconds after its launch, is given by a function f (x). What is the maximum height reached by the missile?

Problem 3 - Given a function f (x) how can you calculate the tangent line to the function graph at one of its points?

These are just three examples of problems whose solution requires the use of a mathematical tool called " derivative":

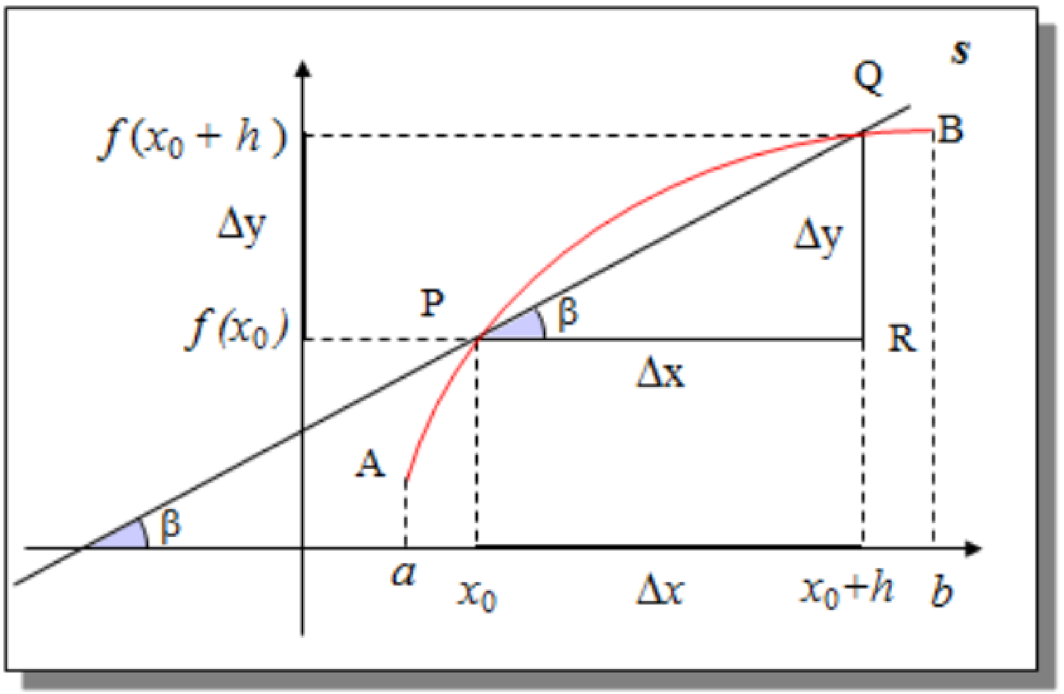
1. The instant variation of a quantity
2. Problems of maximum or minimum, also called optimization problems
3. Calculation of a tangent at any one curve

Intuitively when we speak of "derivative" we need to know how a quantity changes with respect to another,
how dependent variable changes when the independent variable increases or

decreases.
The derivative is closely related to the concept of variation, or better instant variation.

Now we can give a definition of "derivative" starting from the geometric meaning of a derivative.

GEOMETRIC MEANING OF DIFFERENCE QUOTIENT



Let us consider a function f(x) ; given Δx>0 (given Δx greater than zero).

Assume that both the points x0 and x0+Δx lie in the domain of f(x)

Let us consider the two points P (x0, f(x0)) and Q(x0+Δx, f(x0+Δx)) on the Cartesian plane.

The secant to the Graph of the function f(x) is the unique line passing through the two points P and Q.

The slope of the secant is given by the difference quotient

$$\frac{f(x+\Delta x)-f(x)}{(x+\Delta x)-x}=\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

The DIFFERENCE QUOTIENT represents the slope of the straight line s secant curve at the points P and Q, respectively, of abscissas x_0 and x_0+ h

$$\frac{\Delta\gamma}{\Delta x}=m_{PQ}$$

GONIOMETRIC MEANING OF DIFFERENCE QUOTIENT

The DIFFERENCE QUOTIENT represents the goniometric tangent of the β angle that the straight line s (secant at the points of P and Q, respectively, abscissas x_0 and x_0+ h) forms with the positive semi-axis of abscissas.

$$\frac{\Delta y}{\Delta x}=tg\beta$$

GEOMETRIC MEANING OF DERIVATIVE OF A FUNCTION y = f (x)

For a function f(x) at the argument x the derivative is if it exists and is finite, the limit of the difference quotient

$\frac{f(x+\Delta x)-f(x)}{\Delta x}$ when the increment Δx tends to 0. It is written as the

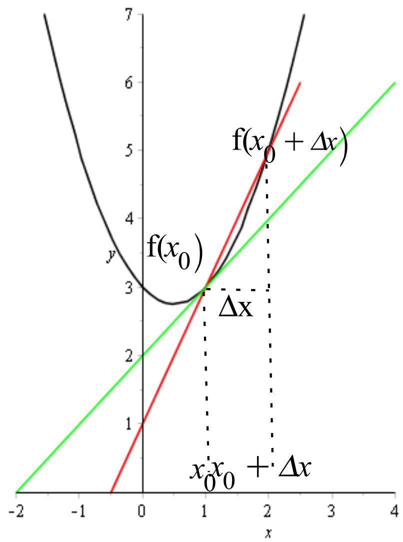
$$\lim_{\Delta x\to 0}\frac{f(x+\Delta x)-f(x)}{\Delta x}$$

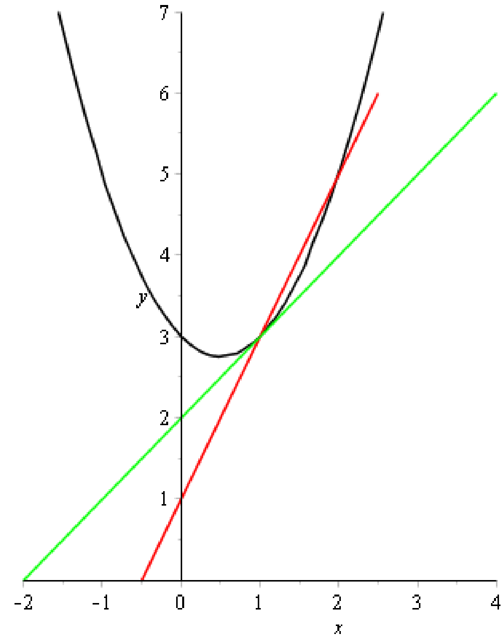
For functions of a single variable, if the left- and right-hand limits exist and are equal, it is the gradient of the curve at x, and is the limit of the gradient of the chord joining the points (x, f(x)) and (x + Δx, f(x + Δx)), as shown, the slope of the tangent to a curve at a point.

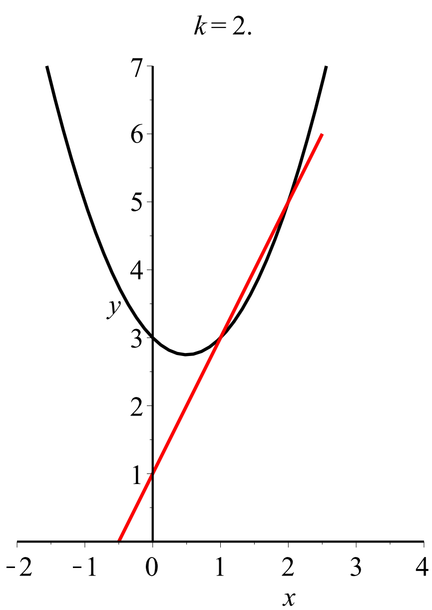
Let us consider $\Delta x = h$, we can write the slope of the tangent to a curve at a point, such us the limit of the difference quotient, when h is approaching 0 :

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \quad (1)$$

▶ costruzione fig 1







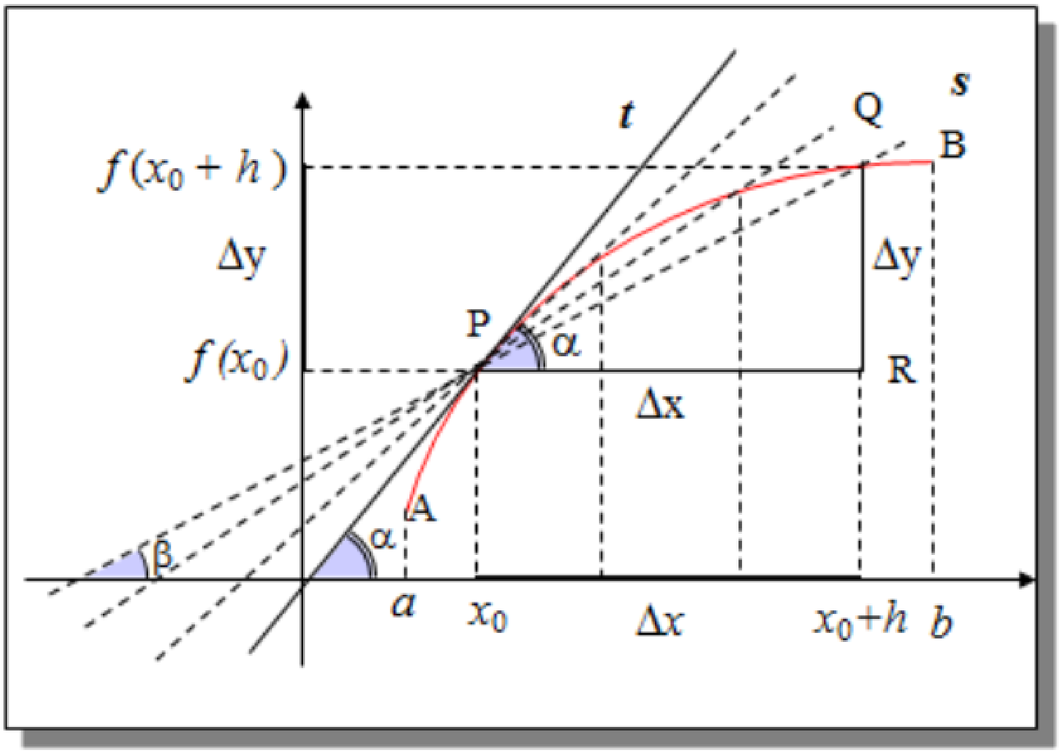
The function of x defined as this limit (1) for each argument x is the first

derivative of y=f(x); it is the rate of change of the value of the function with respect
to the independent variable, and is indicated by one of the equivalent notations:
dy/dx, f'(x), Df(x),
while the ratio of differences of which this is the limit is written Δ y/Δ x.

Therefore, when $\Delta x \to 0$ the secant will be an increasingly good approximation of the tangent at x_0 (see figure).

$$f'(x) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$$

The first derivative of a function y = f (x) at a point x_0 of its domain is the limit (when it exists and is finite) of the DIFFERENCE QUOTIENT when , increase the independent variable, Δx approaching zero and it is indicated by f '(x_0).



if $\Delta x = h$ $$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(x_0+h) - f(x_0)}{h}$$

GEOMETRIC MEANING OF THE FIRST DERIVATIVE

$$\lim_{Q \to P} s_{PQ} = t_P \Rightarrow \lim_{Q \to P} m_{PQ} = m_{t_P} \Rightarrow f'(x_0) = m_{t_P}$$

The first derivative is the slope of the tangent t to the graph of the function at its point P of abscissa x_0.

THE GONIOMETRIC MEANING OF THE FIRST DERIVATIVE

$$\lim_{Q \to P} \beta = \alpha \Rightarrow \lim_{Q \to P} tg\beta = tg\alpha \Rightarrow f'(x_0) = tg\alpha$$

The first derivative is the goniometric tangent of the corner α that the line t, tangent to the the graph of the function in the point P of abscissa x_0, forms with the positive semi-axis of abscissas.

PARTE SECONDA

Support activities for both the input language , and for the output language
(Attività di supporto sia per il linguaggio di input, sia per il linguaggio di output.)

Glossary

Word -level support		Sentence-level support
Word bank Derivative Tangent Secant Graph of the function Cartesian plane Slope Difference quotient corner point inflection point cusp point Maximum	= Derivata = retta tangente = retta secante = grafico della funzione = piano cartesiano = coefficiente angolare = rapporto incrementale = punto angoloso = punto di flesso = cuspide = punto di massimo = punto di minimo = crescente	**Substitution table** **Sentence starters:** **the rate of change of a function** = il tasso di variazione di una funzione **Let us consider** = consideriamo **Come chiedere spiegazioni e chiarimenti -un permesso- offrirsi di fare- attirare l'attenzione** - What does ".............." mean? - How do you say "................" in English? - How do you "Spell

Minimum

Increasing

Decreasing

Concave up

Concave dawn

plus

minus

divided

divided by

Domain
Corner
instant velocity
mean velocity
tangent at a curve
equation
abscissa
nought or zero
all or to
coordinate
ordinate
sine(x)
cosine(x)
less than zero
greater than zero
less than or equal to
greater than or

= decrescente
= concavità verso l'alto
= concavità verso il basso
= +□
= -
= ÷ diviso
= /fratto
= dominio
= angolo
= velocità istantanea
= velocità media
= tangente ad una curva
= equazione
= ascissa
= zero
= elevato
=coordinate
= ordinata
= sin(x)
= cos(x)
= negativo
= positivo
$\leq$
$\geq$
a_n

pronunce" This word?
- Is this corret?
- Is this right?
- Are these ok?
- Is this a mistake?
- Where is this wrong?
- What's wrong with this word/ sentence?
- Is there a difference between ...and....?
- Excuse me, I didn't hear.
- I'm sorry, I don't understand
- Can you say it again, please?
- Can you repeat that, please?
- Can you give an example, explain......please?
- Can you speak more slowly, please?
- Can I open the window, please?
- Can I help(you)?
- Can I clean the blackboard (for you)?
- Do you want a hand with this exercise?
- Can I have "another copy", please ?
- Can I have "an extra sheet", please ?
- I'd like "another copy", please ?
- I'd like "an extra sheet", please ?
- Have you got "another copy",please ?
- Have you got "an extra sheet",please?
- I haven't got a pen. Can someone lend me one?

equal to a sub n		**Come scusarsi** - I'm (terribly sorry), I'm late - I'm (terribly sorry), I've left my book at home - I'm (terribly sorry), I've lost my notebook - I'm (terribly sorry), I haven't done my homework **Come chiedere un'opinione** - Do you like? - What do you think of...? **Come esprimere un'opinione** - I(don't) like...... - I(don't) think that..... **Come esprimere accordo e disaccordo** - I think so too. - I don't think so. - I agree (with you). - I don't agree (with you).

Activities which enable students during the lesson, to move from lower (basso) to higher (alto) order thinking and learning skills

Example:

Lower order thinking questions	Purpose	Higher order thinking questions	Purpose
Application of rules of derivatives 1) What is the derivative of the constant function ? 2) What is the derivative of the function e^x?	To check knowledge (controllare la conoscenza)	1) What is the slope of the line tangent to the curve of equation y= e^x +5 at the point of abscissa x=0 ? 2) Write the equation of the	To develop reasoning and analytical skills (Sviluppare abilità di ragionamento e analitiche)

		tangent to the curve of equation y= e^x+5 at the point of abscissa x=0	

Lower order thinking questions	Purpose	Higher order thinking questions	Purpose
Application of derivation rules 1) What is the derivative of the function y= x-sin (x) ? 2) What is the derivative of the function y= $(2\cdot x-3)^2$?	To check understanding (controllare la comprensione)	1) What is the slope of the line tangent to the curve of equation y= x-sin(x) at the point of abscissa x= 0 ? 2) Write the equation of the tangent to the curve of equation y= x-sin(x) at the point of abscissa x=0	To develop reasoning and analytical skills (Sviluppare abilità di ragionamento e analitiche)

Consolidation Activities / lexical expansion
(Attività di consolidamento/ampliamento lessicale)
Examples:
The Tangent

We found that	the slope of the tangent to a curve at a point	is.............................. because ....
We found that the slope of the tangent to a curve at a point is equal to the first derivative of the function at a point because the tangent is the limit		

position of the secant line

We found that	the graph of the function	is........................... because ………………...

We found that the graph of the function is increasing because the first derivative of the function is greater than zero

We found that the graph of the function is decreasing because the first derivative of the function is less than zero

Physical meaning of the derivative

Velocity

We found that	the instant velocity	is........................... because ………………..

We found that the instant velocity is the first derivative of space with respect to time because the instant velocity is the limit of the mean velocity $v_m = \frac{\Delta s}{\Delta t}$ when Δt is approaching 0 : $v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

Electric current

We found that	the electric current	is........................... because ………………...

We found that the intensity of electric current is an orderly flux of charges.

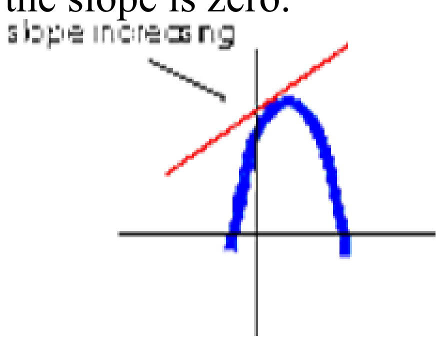
It is defined as the ratio between the amount of charge that passes through the wire and the time t in which this occurs . If the current varies in time, more generally, the current is defined such as the first derivative of charge with respect to time because the intensity of electric current is the limit of the Difference quotient $I = \frac{\Delta q}{\Delta t}$ when Δt is approaching 0 :

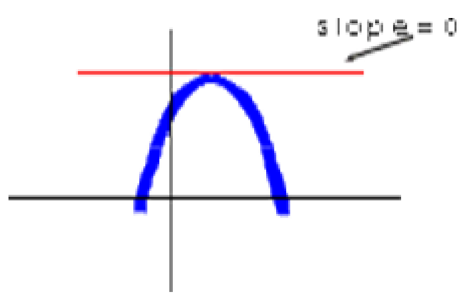
$$I = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

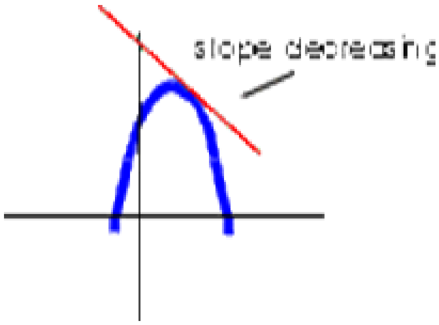
Higher order thinking skills (HOTS)
One of the convenient uses for the derivative of a function is finding the maximum or minimum points of the function.

The derivative indicates the rate of change of a function.
Put in other words, the derivative measures the slope of the function at a particular point.

When the slope changes from positive to negative, the function is at **its maximum** when the slope is zero.
When the slope changes from negative to positive, it is at **its minimum** when the slope is zero.







(We *will provide students with activities so that they can use the knowledge and the skills acquired in the Unit.*
They will develop these skills into higher skills . At the same time students will be given links with other subjects of the curriculum-)

Example:

APPLICATION : Problems related to real life

George had a function of stresses on a particular part of airplane with respect to time in flight. He wanted to determine the time when the part was under the greatest stress.This function is $f(t) = -t^2 + 2t + 3$

Resolution

Given the function
$f(t) = -t^2 + 2t + 3$
George took the derivative of the function
$f'(t) = -2t + 2$
then he determined that the derivative was equal to zero at $t = 1$.
If we consider $-2t+2 > 0$ (greater than zero), we find that for $t < 1$, the first derivative is positive , and for $t > 1$ is negative.
Therefore, the time found of greatest stress was 1 minute after the take off.

DERIVATIVES OF SOME COMMON FUNCTION

DERIVATIVES RULES

⪰ **Derivative of a constant** $D(k) = 0$	**The derivative of a constant is equal to zero.**

≽Derivative of x $D(x) = 1$	The derivative of the function x is equal to one.
≽Derivative of x^n $D(x^n) = n \cdot x^{n-1}$	The derivative of the function x^n (x to the n) is equal to n multiplied by x to the n minus one.
≽ Derivative of x^α $D(x^\alpha) = \alpha \cdot x^{\alpha - 1}$	The derivative of the function x^α (x to the alpha) is equal to alpha multiplied by x all alpha minus one.
≽Derivative of a^x $D(a^x) = a^x \cdot \ln a$	The derivative of exponential function a^x (a all x) is equal to a all x multiplied by natural logarithm of a.
≽Derivative of e^x $D(e^x) = e^x$	The derivative of exponential function e^x (*e* all x) is equal to e all x
≽Derivative of $\log_a(x)$ $D(\log_a(x)) = \frac{1}{x} \cdot \frac{1}{\ln a}$	The derivative of function $\log_a(x)$ (logarithm base *a* of x) is equal to inverse of x multiplied by inverse of natural logarithm of *a*
≽Derivative of $\ln(x)$ D $(\ln(x)) = \frac{1}{x}$	The derivative of function $\ln(x)$(natural logarithm of x) is equal to one divided by x.
≽Derivative of $\sin(x)$ $D(\sin(x))$ $= \cos(x)$	The derivative of function $\sin(x)$ (sine of x)is equal to cosine of x.
	The derivative of function $\cos(x)$ (cosine of x) is equal to minus of

➢Derivative of $\cos(x)$ $D(\cos(x)) = -\sin(x)$	sine of x

By using the derivative rules in combination, we can find the derivatives of many other functions

Here are some basic laws which can be used to derive other differentiation rules.

Derivative of $k \cdot f(x)$ (the product of a constant and a function)

The derivative of the product of a constant and a function of x is equal to the constant multiplied by the first derivative of the function.

$$D[k \cdot f(x)] = k \cdot f'(x)$$

Derivative of the sum of two functions

The derivative of the sum of two functions is equal to the sum of the first derivatives of the functions.

$$D[f(x) + g(x)] = f'(x) + g'(x)$$

Derivative of a product of two functions

The derivative of a product of two functions is equal to the product of the derivative of the first function multiplied by the second function, plus the first function multiplied by the first derivative of the second.

$$D[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Derivative of the reciprocal function

The derivative of the reciprocal function is equal to the opposite of the first derivative of the function divided by the function all squared.

$$D\left[\frac{1}{f(x)}\right] = -\frac{f'(x)}{f^2(x)}$$

Derivative of the quotient of two differentiable functions

The derivative of a fraction, that is, the quotient of two differentiable functions, is equal to the function in the denominator multiplied by the derivative of the function in the numerator, minus the function in the numerator multiplied by derivative of the function in the denominator, all divided by the square of the function in the denominator.

$$D\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)}$$

Theorem of the derivative of a composite function y = f (g (x))

Given the composite function y = f (g (x)) (y equal to f to g x), the derivative of a function f of another function g of x is equal to the first derivative of f with respect to z multiplied by the derivative of g with respect to x.

$$D[f(g(x)]=f'(z)\cdot g'(x)$$

Theorem of the derivative of the inverse of a function

Given a function y = f (x) invertible and differentiable at an interval I, and let
x = F (y) be its inverse function.
At the points where f ' (x) $\neq 0$, the inverse function is differentiable and the derivative of the inverse of a function is equal to the reciprocal of the first derivative of the function given.

$$D[F(y)]=\frac{1}{f'(x)}$$

Derivatives of some inverse functions

Derivative of arcsin(x) $$D[\arcsin(x)]=\frac{1}{\sqrt{1-x^2}}$$ Derivative of arcsin(x) $$D[arcos(x)]=-\frac{1}{\sqrt{1-x^2}}$$	The derivative of arc sine of x is equal to one divided by the square root of the difference between one and the square of x The derivative of arc cosine of x is equal to minus one divided

	by the square root of the difference between one and the square of x
Derivative of arctg(x) $D[arctg(x)] = \frac{1}{1+x^2}$	The derivative of the arctangent of x is equal to one divided by one plus the square of x
Derivative of arccotg(x) $D[\mathrm{arccotg}(x)] = -\frac{1}{1+x^2}$	The derivative of the inverse cotangent of x is equal to minus one divided by one plus the square of x