Allegato 3

Modulo Interdisciplinare CLIL
Titolo del Modulo: “The Derivatives ”
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con il supporto linguistico della prof.ssa Barbara Belfiore

Obiettivi:

1) Conoscere le derivate
2) Saper applicare tale strumento alla risoluzione dei problemi della realtà
3) Migliorare le competenze di "Analisi "
4) Migliorare le competenze della lingua inglese
5) Espandere il patrimonio lessicale in L1 ed L2

Obiettivi Trasversali

1) Saper collaborare con i compagni e partecipare con interesse alle lezioni
2) Saper attivare strategie di apprendimento e apprendere da prospettive diverse
3) Saper riflettere su quanto svolto e individuare punti di forza e criticità
4) Saper attivare i processi cognitivi superiori: distinguere, confrontare, descrivere, sintetizzare, operare collegamenti, etc.

Prerequisiti

Disciplinari	Linguistici
- Il concetto di limite - Il calcolo dei limiti - La continuità delle funzione	- Conoscere le principali strutture linguistiche di livello intermedio. - Capire parole o espressioni scritte e orali inerenti al modulo - Enunciare in forma scritta e orale definizioni e proprietà - Eseguire correttamente istruzioni richieste

Competenze disciplinari

Conoscenze

- Il significato fisico della derivata, il significato geometrico di rapporto incrementale e il significato geometrico di derivata di una funzione in un punto;
- La definizione di derivata in un punto e di funzione derivata;
- L'equazione della retta tangente a una curva;
- Le derivate fondamentali;
- Le regole di derivazione(somma, prodotto, quoziente, derivata funzione composta e delle funzioni inverse)

Abilità

- Saper calcolare la velocità e l'accelerazione istantanea
- Saper calcolare il rapporto incrementale;
- Calcolare l'equazione della retta tangente a una curva,
- Calcolare la derivata di una funzione applicando opportune regole di derivazione;
- Risolvere problemi di ottimizzazione

Competenze linguistiche

- The first aim for the students is to be able to understand the linguistic function giving directions understanding tasks which is used to introduce all the activities they have to carry out.

The expression used are concerned with:
- The imperative (complete, work out, find, explain, prove,…)
- To have to…
- The second purpose is to know and to be able to use the microlanguage used in their activities
- The third objective is to be able to produce the language used to express the rules necessary to work out the activities.

L'attività è stata svolta con metodologia il CLIL.

Tale metodologia prevede un apprendimento fondamentalmente attivo, interazionale e cooperativo diviso in varie fasi .

Fase 1 **INTRODUCTION**
a) attività motivazionale di warming up.
b) attività di verifica dei prerequisiti disciplinari mediante Brainstorming
c) attività di contestualizzazione disciplinare

Fase 2 **READING AND LISTENING**
In questa fase gli studenti hanno lavorato in piccoli gruppi secondo uno svolgimento cooperativo e socializzante

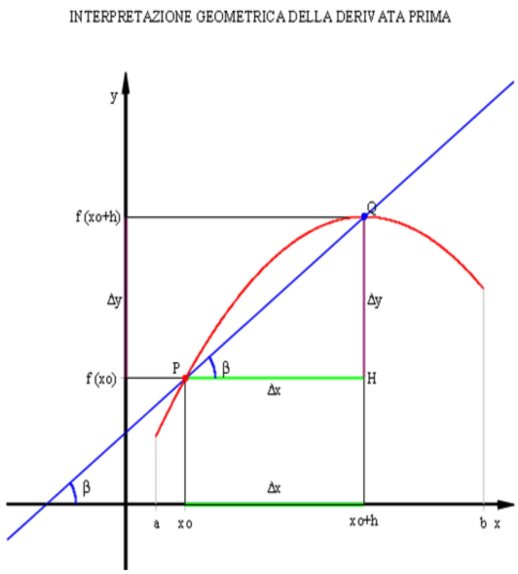
Fase 3 **PRACTICE**
In questa fase sono state proposte attività di consolidamento, rinforzo, approfondimento e verifica in cui gli alunni hanno adoperato le conoscenze e le abilità disciplinari e linguistiche obiettivo del modulo.

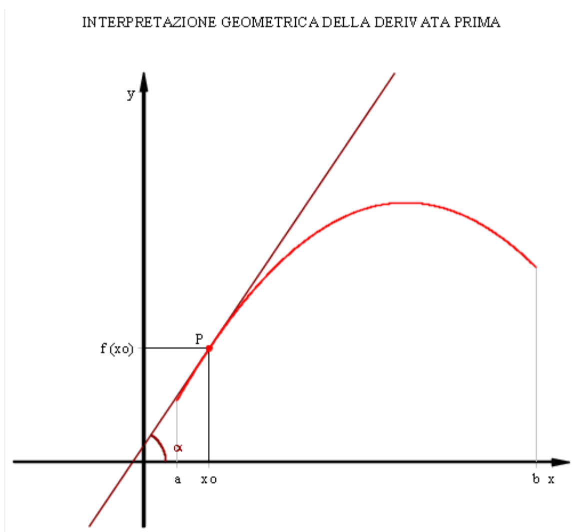
Metodologia (lezione frontale, partecipata, cooperativa, autoformazione,ecc.)
- Lezione frontale
- Problem solving
- Lezioni multimediali di ascolto e visione
- Lavoro a gruppi
- Lezioni cooperative e partecipate

Strumenti (testi, materiali, attività, risorse) -
Schede;
- Lavagna;
- Lavagna multimediale;
- Computer;
- Applicativo Maple
- Piattaforma Moodle

Welcome to the presentation on derivatives.

What is this? Any ideas on this topic?





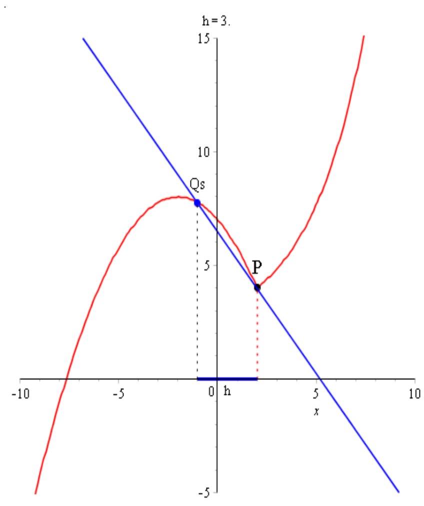
We have two graphs. In these graphs there are straight and curve lines.

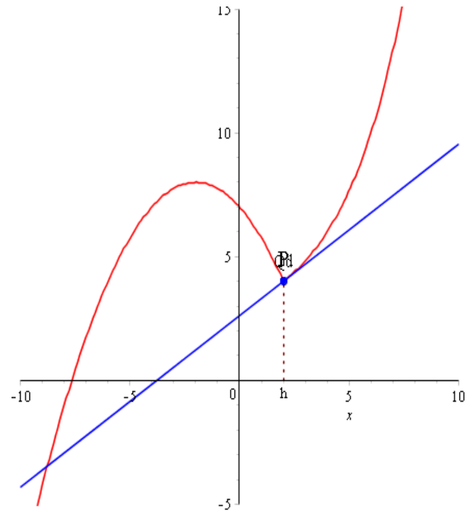
- Which is the difference between the two graphs?
- What is the difference between the two straight lines?

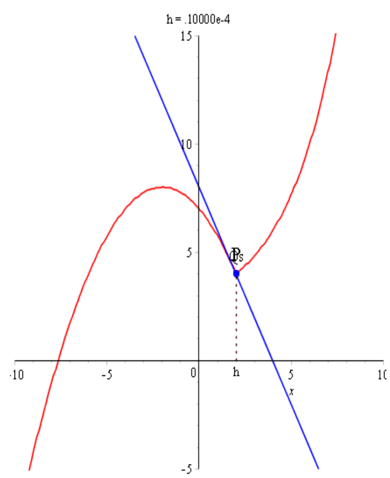
Answer

The first is secant and the second is tangent to the same curve .
The tangent is the limit position of the secant.

Now we see three graphs.







What is the difference in these graphs?
What is the position of the line compared to the curved line?
What is the difference between the first, second and third straight line in these graphs?

Answer

In the first graph the line is secant, in the second and third graphs the lines are tangents to the curve at the same point; but we see two types of

tangents, the slopes of the two tangents exist but are different.
In the second graph Q approaches P from the right while in the third graph Q approaches P from the left
The two tangents are different. Point P is called the *corner point.*

Derivatives and their properties

The focus of this Unit will be the study of derivatives and their properties.
I think that by studying this subject, Maths starts to become a lot more fun than it was just a few topics ago.
Well let's get started with our derivatives.

I know it sounds very complicated, but the derivative rules are very simple.
You don't need to think/reflect.

What is a derivative?
It is a very important mathematical tool.

Where did you find this concept?

1) In the definition of the slope of the tangent to a curve at a point

2) In instant velocity.

3) Whenever we have a difference quotient $\frac{\Delta y}{\Delta x}$ in which Δx approaching 0

By the way, can you give me a definition of "instant velocity"?
It is a change in ………………… and ………………… at a particular point in …………………

The changes …………………at a particular point in a graph.

It is a change in direction and space at a particular point in time

The changes occur at a particular point in a graph.

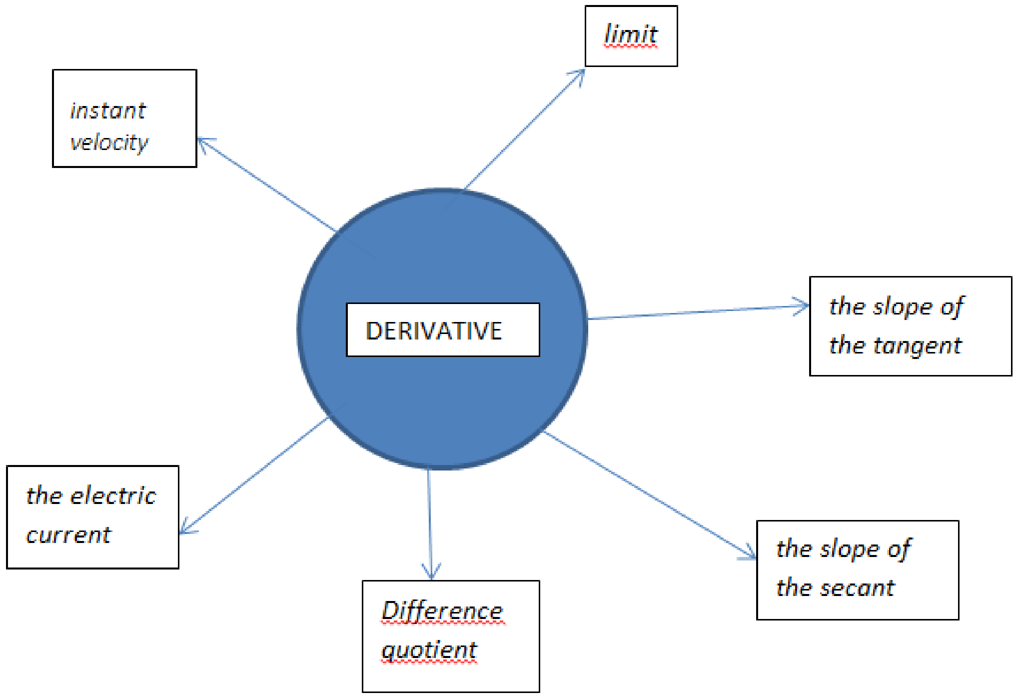
That’s not enough. Can you give me a more precise definition?

It is the limit of .... when ....approaching........

It is the limit of mean velocity when Δt is approaching zero

What is the meaning of the words we find in the following MIND MAP?

MIND MAP



Here is a link to a youtube video on Derivatives:
https://www.youtube.com/watch?v=rAof9Ld5sOg

(Draw on the whiteboard what described in the following passage)

Transcript of the Video on Derivatives

Well, that's my coordinate axes.
In general, if I have a straight line, and I ask you to find the slope,
I think you already know how to do this: it's just the change in y divided by the change in x.

We know that slope is the same across the whole line, but if I want to find the slope at any point in this line,
what I would do is I would pick a point x – say, I'd pick this point.

I could pick any two points, the choice it's pretty arbitrary, and I would figure out what the change in y is.

This is the change in y, delta y, that’s just another way of saying change in y, and this the change in x, delta x.

We figured out that the slope is defined really as change in y divided by change in x.
Another way of saying that is delta y divided by delta x,very straightforward.

Now, what happens, though, if we’re not dealing with a straight line?

Another coordinate axes.

Let’s just say I had the curve y equals x squared .
Let me draw it in a different colour.
So y equals x squared looks something like this.
It’s a curve, you’re probably pretty familiar with it by now.

Now what I’m going to ask you is: what is the slope of this curve?

What does it mean to take the slope of a curve now?
Well, in this line the slope was the same throughout the whole line.
But if you look at this curve, the slope changes, right?
Here it’s almost flat, and it gets steeper steeper steeper steeper steeper until gets pretty steep.

So, you’re probably saying, well, how do you figure out the slope of a curve whose slope keeps changing?

Well there is no slope for the entire curve. For a line, there is a slope for the entire line, because the slope never changes.

But what we could try to do is figure out what the slope is at a given point.
And the slope at a given point would be the same as the slope of a tangent line.

How are we going to figure out what the slope is at any point along the curve y equals x squared?

That’s where the derivative comes into use, and now for the first time you’ll actually see why a limit is actually a useful concept.

Intuitively when we speak of "derivative" we need to know how a quantity changes with respect to another,
how dependent variable changes when the independent variable increases or decreases.
The derivative is closely related to the concept of variation, or better instant variation.

CONTEXTUALIZATION

Problem 1 - A car is travelling along a straight road. The space path varies according to a law of the motion given by a certain function s (t). What is its speed, i.e., how does the space covered vary, instant by instant, with respect to the time taken to travel?

Problem 2 - The height of a missile in metres, t seconds after its launch, is given by a function f (t). What is the maximum height reached by the missile?

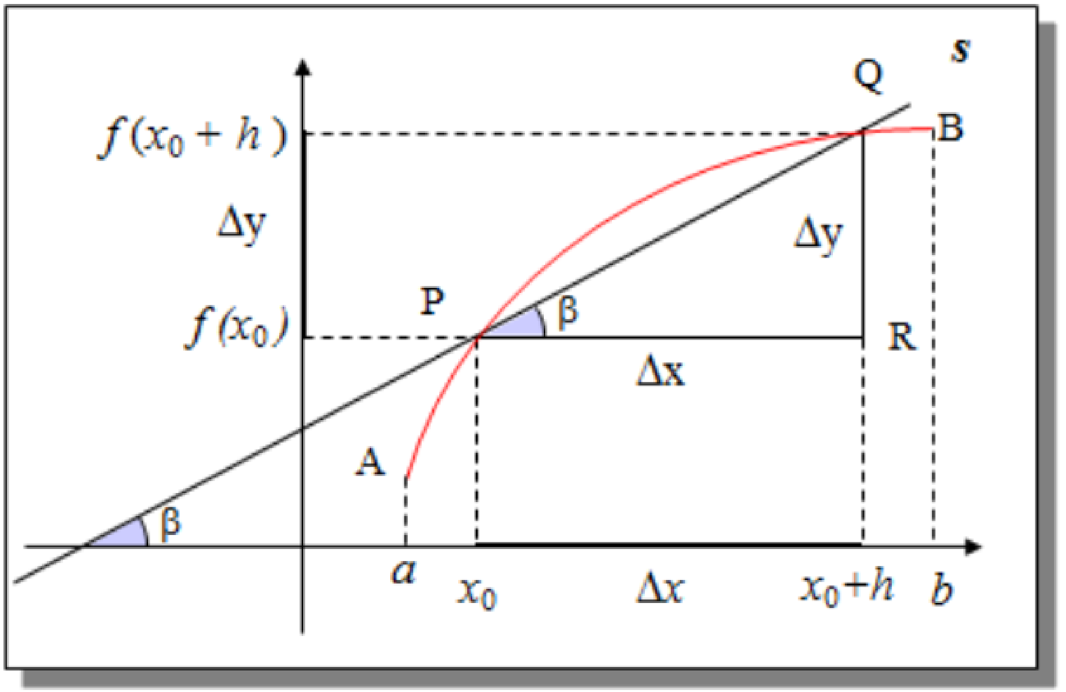
Problem 3 - Given a function f (x) how can you calculate the tangent line to the function graph at one of its points?

These are just three examples of problems whose solution requires the use of a mathematical tool called “ derivative”:

1. The instant variation of a quantity
2. Problems of maximum or minimum, also called optimization problems
3. Calculation of a tangent at any one curve

Intuitively when we speak of "derivative" we need to know how a quantity changes with respect to another, how dependent variable changes when the independent variable increases or decreases. The derivative is closely related to the concept of variation, or better instant variation.
Now we can give a definition of "derivative" starting from the geometric meaning of a derivative.

GEOMETRIC MEANING OF DIFFERENCE QUOTIENT



Let us consider a function f(x) ; given h>0 (given h greater than zero).

Assume that both the points x0 and x0+Δx lie in the domain of f(x)

Let us consider the two points P (x0, f(x0)) and Q(x0+h, f(x0+h)) on the Cartesian plane.

The secant to the Graph of the function f(x) is the unique line passing through the two points P and Q.

The slope of the secant is given by $m = \frac{y - y_0}{x - x_0}$ this is equal to the difference quotient

$$\frac{f(x+h) - f(x)}{(x+h) - x} = \frac{f(x+h) - f(x)}{h}$$

The DIFFERENCE QUOTIENT represents the slope of the straight line s secant curve at the points P and Q, respectively, of abscissas x_0 and x_0+ h

$$\frac{\Delta y}{\Delta x} = m_{PQ}$$

GONIOMETRIC MEANING OF DIFFERENCE QUOTIENT

The DIFFERENCE QUOTIENT (the slope) represents the goniometric tangent of the β angle that the straight line s (secant at the points of P and Q, respectively, abscissas x_0 and x_0+ h) forms with the positive semi-axis of abscissas.

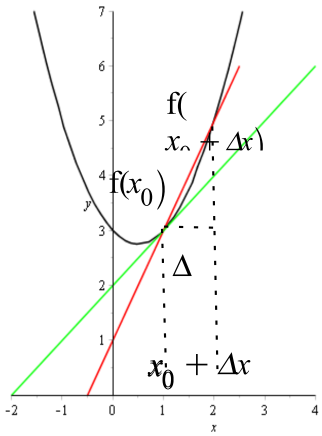
$$\frac{\Delta y}{\Delta x} = tg\beta$$

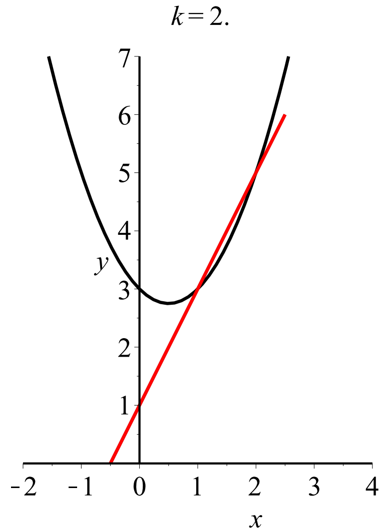
DERIVATIVE OF A FUNCTION y = f (x)

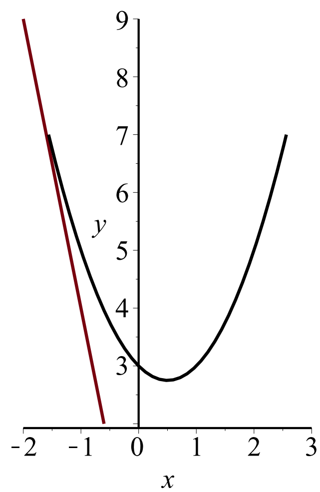
For a function f(x) at the argument x the derivative is, if it exists and is finite, the limit of the difference quotient $\frac{f(x+\Delta x)-f(x)}{\Delta x}$ when the increment Δx approaching to 0.

It is written as the $\lim_{\Delta x \to 0} \frac{f(x+\Delta x)-f(x)}{\Delta x}$

► **costruzione fig 1**







The function of x defined as $\lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ for each argument x is the first derivative of y=f(x); it is the rate of change of the value of the function with respect to the independent variable, and is indicated by one of the equivalent notations:

dy/dx, f'(x), Df(x),

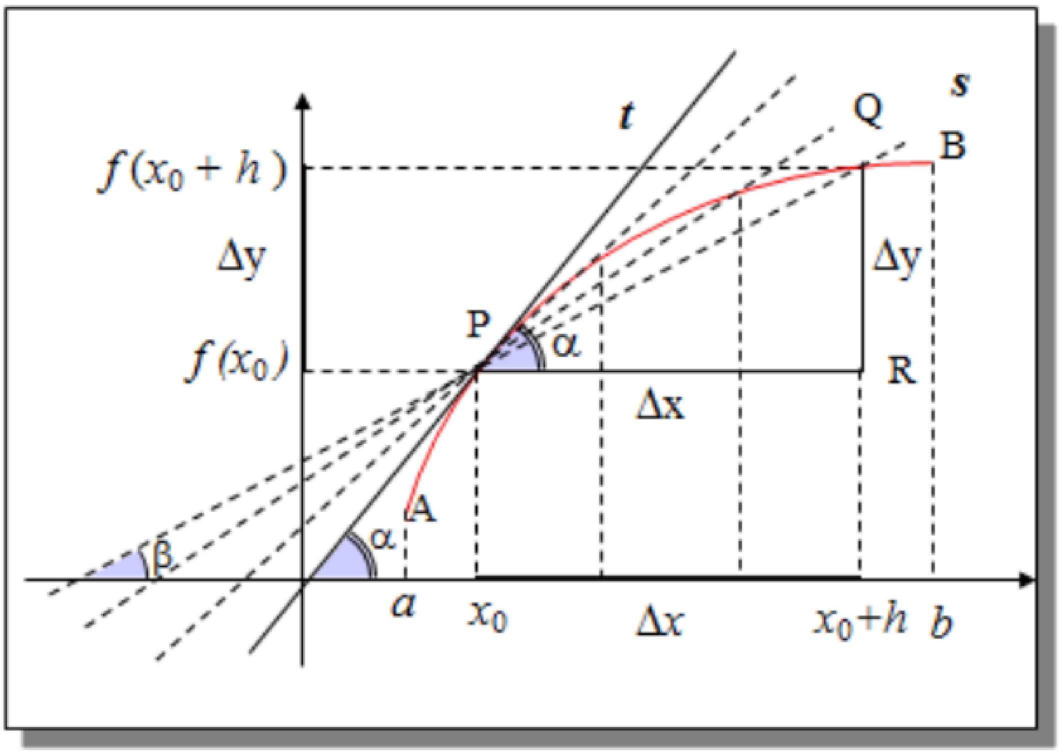
while the ratio of differences of which this is the limit is written Δ y/Δ x.

Therefore, when

$\Delta x \to 0$ the secant will be an increasingly good approximation of the tangent at x_0 (see figure)

So $f'(x) = \lim_{\Delta x \to 0} \dfrac{\Delta y}{\Delta x} = \lim_{\Delta x \to 0} \dfrac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$

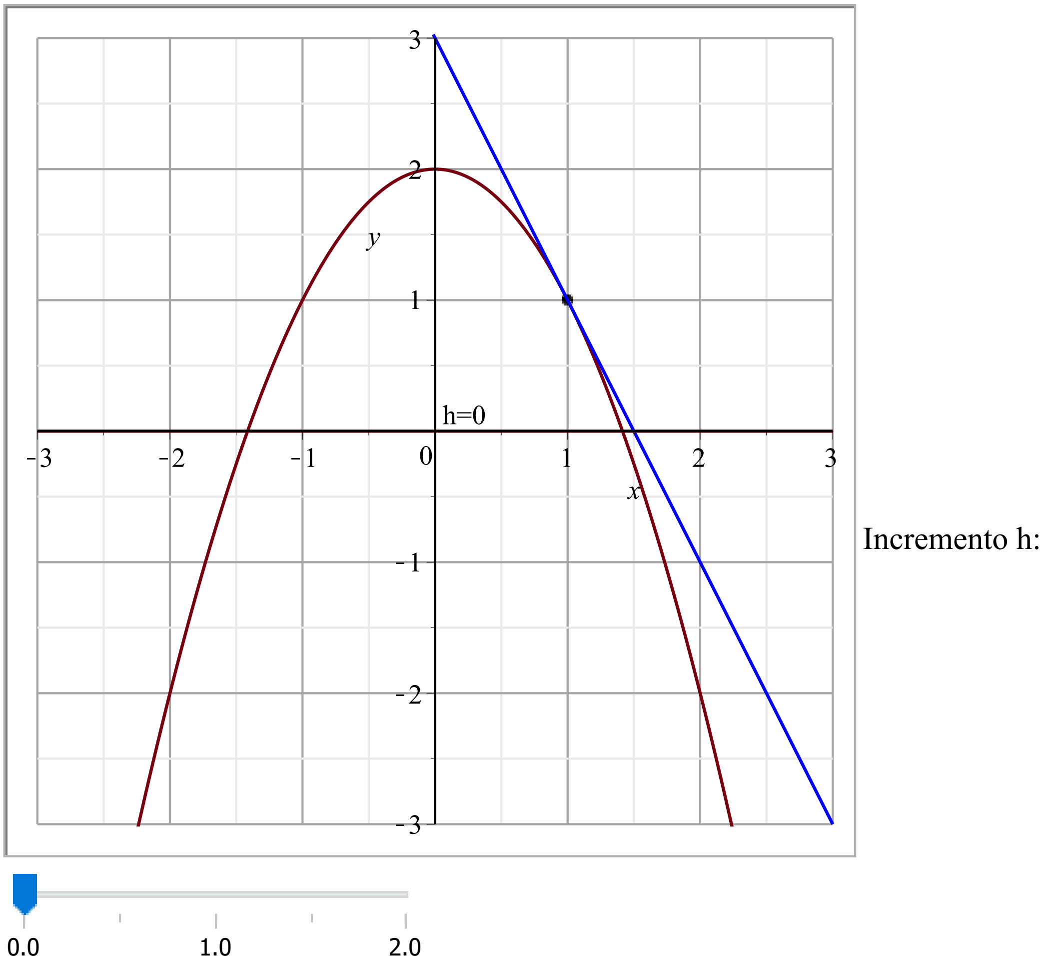
The first derivative of a function y = f (x) at a point x_0 of its domain is the limit (when it exists and is finite) of the DIFFERENCE QUOTIENT when , increase the independent variable, Δx approaching zero and it is indicated by f '(x_0).



if $\Delta x = h$

$$f'(x_0) = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{f(x_0 + h) - f(x_0)}{h}$$

Inizializza $x_0 =$ 1



GEOMETRIC MEANING OF THE FIRST DERIVATIVE

$$\lim_{Q \to P} s_{PQ} = t_P \Rightarrow \lim_{Q \to P} m_{PQ} = m_{t_P} \Rightarrow \boldsymbol{f'(x_0)} = m_{t_P}$$

For functions of a single variable, if the left- and right-hand limits exist and are equal, it is the gradient of the curve at x, and is the limit of the gradient of the chord joining the points (x, f(x)) and (x + Δx, f(x + Δx)), as shown, the slope of the tangent to a curve at a point.

Let us consider $\Delta x = h$, we can write the slope of the tangent to a curve at a point, such us the limit of the difference quotient, when h is approaching 0 :

$$m = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Definition of the GEOMETRIC MEANING

The first derivative is the slope of the tangent to the graph of the function at its point P of abscissa x_0 .

THE GONIOMETRIC MEANING OF THE FIRST DERIVATIVE

$$\lim_{Q \to P} \beta = \alpha \;\Rightarrow \lim_{Q \to P} tg\beta = tg\alpha \;\Rightarrow \boldsymbol{f'(x_0)} = tg\alpha$$

The first derivative is the goniometric tangent of the corner α that the line t, tangent to the graph of the function in the point P of abscissa x_0, forms with the positive semi-axis of abscissas.

DERIVATIVES RULES AND DEMOSTRATIONS

Derivative of a constant $\mathbf{D}(k) = 0$

La derivata della funzione costante y=k è zero

The derivative of a constant is equal to zero.

DIM

$y = f(x) = k$

$\Delta y = k - k = 0$

$\Delta x = h$

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{0}{h} = 0$$

Derivative of x $\mathbf{D}(x) = 1$

La derivata della funzione y=x è 1

The derivative of the function x is equal to one.

DIM

$y = f(x) = x$

$\Delta y = f(x + h) - f(x) = \mathrm{x} + \mathrm{h} - \mathrm{x} = \mathrm{h}$

$\Delta x = h$

$$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{\mathrm{h}}{h} = 1$$

Derivative of x^n $\mathbf{D}(x^n) = n \cdot x^{n-1}$

La derivata della funzione y=x^n è $n \cdot x^{n-1}$

The derivative of the function $\boldsymbol{x}^{\boldsymbol{n}}$ (x to n) is equal to n multiplied by x to the n minus one.

DIM

$y = f(x) = x^n$

$\Delta y = f(x+h) - f(x) = (x+h)^n - x^n$

$\Delta x = h$

$$\# \, y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{(x+h)^n - x^n}{h} = \textit{raccogliendo a fattor comune} \quad x^n$$

$$\lim_{h\to 0} x^n \cdot \frac{\left(\frac{x+h}{x}\right)^n - 1}{h} = \lim_{h\to 0} x^n \cdot \frac{\left(1+\frac{h}{x}\right)^n - 1}{h}$$

$$= \textit{moltiplicando e dividendo per } x \quad \lim_{h\to 0} x^n \cdot \frac{\left(1+\frac{h}{x}\right)^n - 1}{\frac{h}{x}} \cdot \frac{1}{x} =$$

$$\lim_{h\to 0} x^{n-1} \cdot \frac{\left(1+\frac{h}{x}\right)^n - 1}{\frac{h}{x}} = \textit{applicando il limite notevole} \quad \lim_{h\to 0} \frac{\left(1+\frac{h}{x}\right)^n - 1}{\frac{h}{x}}$$

$$= \textit{n e sostituendo} \quad \textit{si ha che} \quad \lim_{h\to 0} x^{n-1} \cdot \frac{\left(1+\frac{h}{x}\right)^n - 1}{\frac{h}{x}} = x^{n-1} \cdot n$$

esempio

#D$(x^2) = 2 \cdot x$

#D$(x^5) = 5 \cdot x^4$

Derivative of x^{α} $D(x^{\alpha}) = \alpha \cdot x^{\alpha - 1}$

La derivata della funzione y=x^{α} è $\alpha \cdot x^{\alpha-1}$

The derivative of the function $\boldsymbol{x}^{\boldsymbol{\alpha}}$ (x to alpha) is equal to alpha multiplied by x all alpha minus one.

Quanto dimostrato per l'esponente intero vale anche quando l'esponente è frazionario

Sappiamo che una potenza con esponente frazionario è una radice , quindi per eseguire le derivate dei radicali si opera eseguendo le seguenti operazioni.

$diff(\mathrm{sqrt}(x), x)$

$$\frac{1}{2\sqrt{x}} \qquad \textbf{(12.1)}$$

$\sqrt[5]{x^3} = x^{\frac{3}{5}}$

$D\left(\sqrt[5]{x^3}\right) = \mathrm{D}\left(x^{\frac{3}{5}}\right) = \frac{3}{5} \cdot x^{\frac{3}{5}-1} = \frac{3}{5} \cdot x^{\frac{3-5}{5}} = \frac{3}{5} \cdot x^{\frac{-2}{5}} = \frac{3}{5\sqrt[5]{x^2}}$

Derivative of a^x $\mathbf{D}(a^x) = a^x \cdot \ln a$

La derivata della funzione y= a^x è $a^x \cdot \ln a$

The derivative of exponential function a^x (a all x) is equal to a all x multiplied by natural logarithm of a.

DIM

$y = f(x) = a^x$

$\Delta y = f(x+h) - f(x) = a^{x+h} - a^x$

$\Delta x = h$

$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{a^{x+h} - a^x}{h} = \lim_{h \to 0} \frac{a^x \cdot a^h - a^x}{h}$ = *raccogliendo a fattor comune* a^x

$\lim_{h \to 0} a^x \cdot \frac{a^h - 1}{h}$ = *per il limite notevole*

$\lim_{h \to 0} \frac{a^h - 1}{h} = \ln a$ *sostituendo* *si ha che* $\lim_{h \to 0} a^x \cdot \frac{a^h - 1}{h} = a^x \cdot \ln a$

esempio

#D$(3^x) = 3^x \cdot \ln 3$

Derivative of e^x $\mathrm{D}(e^x) = e^x$

La derivata della funzione y= e^x è e^x

The derivative of exponential function e^x (*e* all x) is equal to e all x

DIM

Sappiamo che $\mathbf{D}(a^x) = a^x \cdot \ln a$

sostituendo la base si ha

$\mathbf{D}(e^x) = e^x \cdot \ln e = e^x \cdot 1 = e^x$

Derivative of $\log_a(x)$ $\quad D(\log_a(x)) = \frac{1}{x} \cdot \frac{1}{\ln a}$

La derivata della funzione y= $\log_a(x)$ **è** $\frac{1}{x} \cdot \frac{1}{\ln a}$

The derivative of function $\log_a(x)$ (logarithm base *a* of x) is equal to inverse of x multiplied by inverse of natural logarithm of *a*

DIM

$y = f(x) = \log_a(x)$

$\Delta y = f(x+h) - f(x) = \log_a(x+h) - \log_a(x)$

$\Delta x = h$

$y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{\log_a(x+h) - \log_a(x)}{h}$ = per le proprietà dei logaritmi

$\lim_{h \to 0} \frac{\log_a\left(\frac{x+h}{x}\right)}{h} = \lim_{h \to 0} \frac{\log_a\left(1 + \frac{h}{x}\right)}{h}$ *moltiplicando e dividendo per x si ha*

$\lim_{h \to 0} \frac{\log_a\left(1 + \frac{h}{x}\right)}{\frac{h}{x} \cdot x}$ *per il limite notevole*

$\lim_{h \to 0} \frac{\log_a\left(1 + \frac{h}{x}\right)}{\frac{h}{x}} = \log_a(e) = \frac{1}{\ln a}$ *sostituendo* *si ha che*

$\lim_{h \to 0} \frac{\log_a\left(1 + \frac{h}{x}\right)}{\frac{h}{x}} \cdot \frac{1}{x} = \frac{1}{x \cdot} \frac{1}{\ln a}$

esempio

#D$(\log_5(x)) = \frac{1}{x} \cdot \frac{1}{\ln 5}$

Derivative of $\ln(x)$ $\quad D(\ln(x)) = \frac{1}{x}$

La derivata della funzione y= $\ln(x)$ è $\frac{1}{x}$

The derivative of function $\ln(x)$(natural logarithm of x) is equal to one divided by x.

DIM

Sappiamo che $\mathbf{D}\left(\log_a(x)\right) = \frac{\mathbf{1}}{x} \cdot \frac{1}{\ln a}$

sostituendo la base si ha

$$\mathbf{D}\left(\log_e(x)\right) = \frac{\mathbf{1}}{x} \cdot \frac{1}{\ln e} = \frac{1}{x} \cdot 1 = \frac{1}{x}$$

Derivative of $\sin(x)$ $\mathbf{D}(\sin(x)) = \cos(x)$

La derivata della funzione y= è $\cos(x)$

The derivative of function $\sin(x)$ (sine of x)is equal to cosine of x.

DIM

$y = f(x) = \sin(x)$

$\Delta y = f(x+h) - f(x) = \sin(x+h) - \sin(x)$

$\Delta x = h$

$$\# \; y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{\sin(x+h) - \sin(x)}{h} = \text{applicando le formule di prostaferesi}$$

$$\lim_{h \to 0} \frac{2\cos\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h} =$$

$$\lim_{h \to 0} \frac{\cos\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \qquad \textit{per il limite notevole } \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} = 1 \textit{ si ha che}$$

$$\lim_{h \to 0} \frac{\cos\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} = \cos(x)$$

Derivative of $\cos(x)$ $\mathbf{D}(\cos(x)) = -\sin(x)$

La derivata della funzione y= cos(x) è -sin(x) .

The derivative of function $\cos(x)$ (cosine of x) is equal to minus of sine of x

DIM

$y = f(x) = \cos(x)$

$\Delta y = f(x+h) - f(x) = \cos(x+h) - \cos(x)$

$\Delta x = h$

$$\# \, y' = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x} = \lim_{h \to 0} \frac{\cos(x+h) - \cos(x)}{h} = \text{applicando le formule di prostaferesi}$$

$$\lim_{h \to 0} \frac{-2\sin\left(\frac{x+h+x}{2}\right)\sin\left(\frac{x+h-x}{2}\right)}{h} =$$

$$\lim_{h \to 0} \frac{-\sin\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} \quad \textit{per il limite notevole} \quad \lim_{h \to 0} \frac{\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} = 1 \textit{ si ha che}$$

$$\lim_{h \to 0} \frac{-\sin\left(x+\frac{h}{2}\right)\sin\left(\frac{h}{2}\right)}{\frac{h}{2}} = -\sin(x)$$

Derivative of the product of a constant and a function

La derivata del prodotto di un numero reale per una funzione è uguale al prodotto del numero reale per la derivata della funzione

$$\mathbf{D}[k \cdot f(x)] = k \cdot f'(x)$$

The derivative of the product of a constant and a function of x is equal to the constant multiplied by the first derivative of the function.

$$\mathbf{D}[k \cdot f(x)] = k \cdot f'(x)$$

Esempi

Modalità Worksheet

```
> restart;
  f := x→sin(x);
  k := 2;
  Diff(f(x), x) = diff(f(x), x);
  Diff(k·f(x), x) = diff(k·f(x), x);
```

$$f := x \rightarrow \sin(x)$$

$$k := 2$$

$$\frac{d}{dx} \sin(x) = \cos(x)$$

$$\frac{d}{dx} (2\sin(x)) = 2\cos(x) \qquad \textbf{(19.1.1.1)}$$

```
>
```

Modalità interattiva

Dimostrazione

Derivative of the sum of two functions

La derivata della somma di due funzioni è uguale alla derivata della somma delle derivate delle singole funzioni

$$\mathbf{D}[f(x)+g(x)]=f'(x)+g'(x)$$

The derivative of the sum of two functions is equal to the sum of the first derivatives of the functions.

$$\mathbf{D}[f(x)+g(x)]=f'(x)+g'(x)$$

Esempi

Dimostrazione

Si ha successivamente:

$$\mathbf{D}[f(x)+g(x)] = \lim_{h \to 0} \frac{[f(x+h)+g(x+h)]-[f(x)+g(x)]}{h} =$$

$$\lim_{h \to 0} \frac{[f(x+h)-f(x)]+[g(x+h)-g(x)]}{h} = \lim_{h \to 0}\left[\frac{f(x+h)-f(x)}{h} + \frac{g(x+h)-g(x)}{h}\right] = \lim_{h \to 0} \frac{f(x+h)-f(x)}{h} + \lim_{h \to 0} \frac{g(x+h)-g(x)}{h} \Bigg] =$$

$$f'(x)+g'(x)$$

Derivative of a product of two functions

La derivata del prodotto di due funzioni derivabili è uguale alla somma tra il prodotto della derivata della prima funzione per la seconda non derivata e il prodotto della prima funzione non derivata per la derivata delle seconda.

$$\mathbf{D}[f(x)\cdot g(x)]=f'(x)\cdot g(x)+f(x)\cdot g'(x)$$

The derivative of a product of two functions is equal to the product of the derivative of the first function multiplied by the second function, plus the first function multiplied by the first derivative of the second.

$$\mathbf{D}[f(x)\cdot g(x)]=f'(x)\cdot g(x)+f(x)\cdot g'(x)$$

Esempi

Dimostrazione

.

Si ha successivamente:

$$\mathbf{D}[\boldsymbol{f}(\boldsymbol{x})\cdot\mathbf{g}(\boldsymbol{x})] = \lim_{h\to 0}\frac{f(x+h)\cdot g(x+h)-f(x)\cdot g(x)}{h}$$ = **aggiungiamo e sottraiamo al numeratore**

\# $g(x+h)\cdot f(x)$

$$\lim_{h\to 0}\frac{f(x+h)\cdot g(x+h) - g(x+h)\cdot f(x) + g(x+h)\cdot f(x) - f(x)\cdot g(x)}{h} =$$

$$\lim_{h\to 0}\frac{g(x+h)\cdot[f(x+h)-f(x)]+f(x)\cdot[g(x+h)-g(x)]}{h} =$$

$$\lim_{h\to 0}\frac{g(x+h)\cdot[f(x+h)-f(x)]}{h}+\lim_{h\to 0}\frac{f(x)\cdot[g(x+h)-g(x)]}{h} =$$

$$\lim_{h\to 0}g(x+h)\cdot\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}+\lim_{h\to 0}f(x)\cdot\lim_{h\to 0}\frac{[g(x+h)-g(x)]}{h} =$$

$$g(x)\cdot f'(x)+f(x)\cdot g'(x) = \boldsymbol{f}'(\boldsymbol{x})\cdot\boldsymbol{g}(\boldsymbol{x})+\boldsymbol{f}(\boldsymbol{x})\cdot\boldsymbol{g}'(\boldsymbol{x})$$

Derivative of the reciprocal function

La derivata della funzione reciproca è data da una frazione che ha al numeratore l'opposto della derivata e a denominatore il quadrato della funzione.

$$\mathbf{D}\left[\frac{1}{f(x)}\right]=-\frac{f'(x)}{f^2(x)}$$

The derivative of the reciprocal function is equal to the opposite of the first derivative of the function divided by the function all squared.

$$\mathbf{D}\left[\frac{1}{f(x)}\right]=-\frac{f'(x)}{f^2(x)}$$

Esempi

Dimostrazione

Si ha successivamente:

$$D\left[\frac{1}{f(x)}\right] = \lim_{h\to 0}\frac{\frac{1}{f(x+h)}-\frac{1}{f(x)}}{h} = \lim_{h\to 0}\frac{\frac{f(x)-f(x+h)}{f(x+h)\cdot f(x)}}{h} =$$

$$\lim_{h\to 0}\left[\frac{f(x)-f(x+h)}{f(x+h)\cdot f(x)}\cdot\frac{1}{h}\right] = -\lim_{h\to 0}\left[\frac{f(x+h)-f(x)}{h}\cdot\frac{1}{f(x+h)\cdot f(x)}\right] = -$$

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}\cdot\lim_{h\to 0}\frac{1}{f(x+h)\cdot f(x)} = -f'(x)\cdot\frac{1}{f^2(x)} = -\frac{f'(x)}{f^2(x)}$$

Derivative of the quotient of two differentiable functions

La derivata del quoziente di due funzioni derivabili è una frazione il cui numeratore è dato dalla differenza fra la derivata del numeratore moltiplicata per il denominatore e il numeratore moltiplicato per la derivata del denominatore e il cui denominatore è uguale al quadrato del denominatore.

$$D\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}$$

The derivative of a fraction, that is, the quotient of two differentiable functions, is equal to the function in the denominator multiplied by the derivative of the function in the numerator, minus the function in the numerator multiplied by derivative of the function in the denominator, all divided by the square of the function in the denominator.

$$D\left[\frac{f(x)}{g(x)}\right] = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}$$

Esempi

Dimostrazione

$$D\left[\frac{f(x)}{g(x)}\right] = D\left[f(x)\cdot\frac{1}{g(x)}\right]$$ applichiamo adesso la formula della derivata di un prodotto

$$= Df(x)\cdot\frac{1}{g(x)} + f(x)\cdot D\left[\frac{1}{g(x)}\right] = f'(x)\cdot\frac{1}{g(x)} + f(x)\cdot\left[-\frac{g'(x)}{g^2(x)}\right] =$$

$$\frac{f'(x)}{g(x)} - \frac{f(x)\cdot g'(x)}{g^2(x)} = \frac{f'(x)\cdot g(x) - f(x)\cdot g'(x)}{g^2(x)}$$

Theorem of the derivative of a composite function $y = f(g(x))$

Data una funzione z=g(x) derivabile in x e sia y=f(z) una funzione derivabile in z=g(x), allora la funzione composta
y=F(x)=f(g(x)) è derivabile in x e la sua derivata è il prodotto della derivata di f rispetto a z per la derivata di g rispetto a x.

$$\mathbf{D}[\mathbf{f}(\mathbf{g}(\mathbf{x})] = f'(z) \cdot g'(x)$$

Given the composite function y = f (g (x)) (y equal to f to g x), the derivative of a function f of another function g of x is equal to the first derivative of f with respect to z multiplied by the derivative of g with respect to x.

$$\mathbf{D}[\mathbf{f}(\mathbf{g}(\mathbf{x})] = f'(z) \cdot g'(x)$$

Theorem of the derivative of the inverse of a function

Data una funzione y=f(x) invertibile e derivabile in un intervallo I e sia x=F(y) la sua funzione inversa. Nei punti in cui f '(x) ≠ 0, la funzione inversa è derivabile e la sua derivata è il reciproco della derivata della funzione data .

$$\mathbf{D}[\mathbf{F}(\mathbf{y})] = \frac{\mathbf{1}}{f'(x)}$$

Given a function y = f (x) invertible and differentiable at an interval I, and let x = F (y) be its inverse function.
At the points where f ' (x) ≠ 0 , the inverse function is differentiable and the derivative of the inverse of a function is equal to the reciprocal of the first derivative of the function given.

$$\mathbf{D}[\mathbf{F}(\mathbf{y})] = \frac{\mathbf{1}}{f'(x)}$$

SECOND PART

Support activities for both the input language , and for the output language
(Attività di supporto sia per il linguaggio di input, sia per il linguaggio di output.)

Glossary

Word -level support		Sentence-level support
Word bank Derivative Tangent Secant Graph of the function Cartesian plane Slope Difference quotient corner point inflection point cusp point Maximum Minimum Increasing	 = Derivata = retta tangente = retta secante = grafico della funzione = piano cartesiano = coefficiente angolare = rapporto incrementale = punto angoloso = punto di flesso = cuspide = punto di massimo = punto di minimo = crescente = decrescente = concavità verso l'alto = concavità verso il basso = +□ = -	**Substitution table** **Sentence starters:** **the rate of change of a function** = il tasso di variazione di una funzione **Let us consider** = consideriamo **Come chiedere spiegazioni e chiarimenti -un permesso- offrirsi di fare- attirare l'attenzione** - What does ".............." mean? - How do you say "................" in English? - How do you "Spell pronunce" This word? - Is this correct? - Is this right? - Are these ok? - Is this a mistake? - Where is this wrong? - What's wrong with this

Decreasing Concave up Concave down plus minus divided divided by Domain Corner instant velocity mean velocity tangent at a curve equation abscissa nought or zero all or to coordinate ordinate sine(x) cosine(x) less than zero greater than zero less than or equal to greater than or equal to a sub n rate of change equation of motion Logarithm Power Root	= ÷ diviso = /fratto = dominio = angolo = velocità istantanea = velocità media = tangente ad una curva = equazione = ascissa = zero = elevato =coordinate = ordinata = sin(x) = cos(x) = negativo = positivo $\leq$ $\geq$ a_n tasso di variazione legge oraria del moto Logaritmo Potenza radice radice quadrata radice cubica reciproco bisettrice equazione binomia m.c.m MCD Raccoglimento a fattor comune numeratore	word/ sentence? - Is there a difference between ...and....? - Excuse me, I didn't hear. - I'm sorry, I don't understand - Can you say it again, please? - Can you repeat that, please? - Can you give an example, explain......please? - Can you speak more slowly, please? - Can I open the window, please? - Can I help(you)? - Can I clean the blackboard (for you)? - Do you want a hand with this exercise? - Can I have "another copy", please ? - Can I have "an extra sheet", please ? - I'd like "another copy", please ? - I'd like "an extra sheet", please ? - Have you got "another copy",please ? - Have you got "an extra sheet",please? - I haven't got a pen. Can someone lend me one? **Come scusarsi** - I'm (terribly sorry), I'm late - I'm (terribly sorry), I've left my book at home - I'm (terribly sorry), I've lost my notebook - I'm (terribly sorry), I haven't done my homework **Come chiedere un'opinione**

square root cube roote reciprocal bisector binomial equation lowest common multiple highest common factor Etraction de greatest common factor nominator denominator Putting together properly	denominatore raggruppando opportunamente	- Do you like? - What do you think of...? **Come esprimere un'opinione** - I(don't) like...... - I(don't) think that..... **Come esprimere accordo e disaccordo** - I think so too. - I don't think so. - I agree (with you). - I don't agree (with you).

Activities which enable students during the lesson, to move from lower (basso) to higher (alto) order thinking and learning skills

Example:

Lower order thinking questions	Purpose (scopo)	Higher order thinking questions	Purpose
Application of rules of derivatives 1) What is the derivative of the constant function ? 2) What is the derivative of the function e^x?	To check knowledge (controllare la conoscenza)	1) What is the slope of the line tangent to the curve of equation $y= e^x+5$ at the point of abscissa x=0 ? 2) Write the equation of the tangent to the curve of equation $y= e^x+5$ at the point of abscissa x=0	To develop reasoning and analytical skills (Sviluppare abilità di ragionamento e analitiche)

Lower order thinking questions	Purpose	Higher order thinking questions	Purpose
Application of derivation rules 1) What is the derivative of the function y= x-sin (x) ? 2) What is the derivative of the function y= $(2 \cdot x-3)^2$?	To check understanding (controllare la comprensione)	1) What is the slope of the line tangent to the curve of equation y= x-sin(x) at the point of abscissa x=0 ? 2) Write the equation of the tangent to the curve of equation y= x-sin(x) at the point of abscissa x=0	To develop reasoning and analytical skills (Sviluppare abilità di ragionamento e analitiche)

Consolidation Activities / lexical expansion
(Attività di consolidamento/ampliamento lessicale)
Examples:
The Tangent

We found that	the slope of the tangent to a curve at a point	is.............................. because ………………...
We found that the slope of the tangent to a curve at a point is equal to the first derivative of the function at a point because the tangent is the limit position of the secant line		

We found that	the graph of the function	is............................... because ………………...

We found that the graph of the function is increasing because the first derivative of the function is greater than zero

We found that the graph of the function is decreasing because the first derivative of the function is less than zero

Physical meaning of the derivative

Velocity

We found that	the instant velocity	is............................ because ....

We found that the instant velocity is the first derivative of space with respect to time because the instant velocity is the limit of the mean velocity $v_m = \dfrac{\Delta s}{\Delta t}$ when Δt is approaching 0 : $v = \lim\limits_{\Delta t \to 0} \dfrac{\Delta s}{\Delta t} = \dfrac{ds}{dt}$

Electric current

We found that	the electric current	is............................ because ....

We found that the intensity of electric current is an orderly flux of charges.
It is defined as the ratio between the amount of charge that passes through the wire and the time t in which this occurs . If the current varies in time, more generally, the current is defined such as the first derivative of charge with respect to time because the intensity of electric current is the limit of the Difference quotient $I = \dfrac{\Delta q}{\Delta t}$ when Δt is approaching 0 :

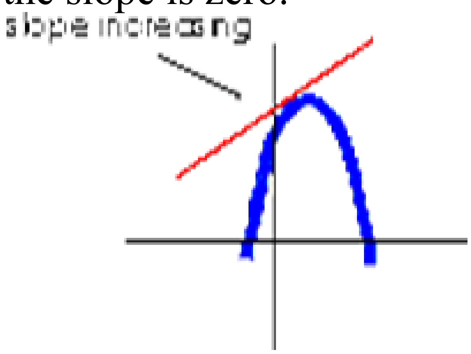
$$I = \lim_{\Delta t \to 0} \frac{\Delta q}{\Delta t} = \frac{dq}{dt}$$

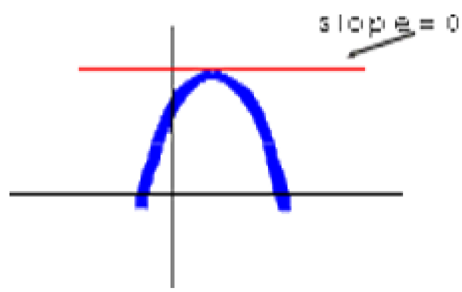
Higher order thinking skills (HOTS)

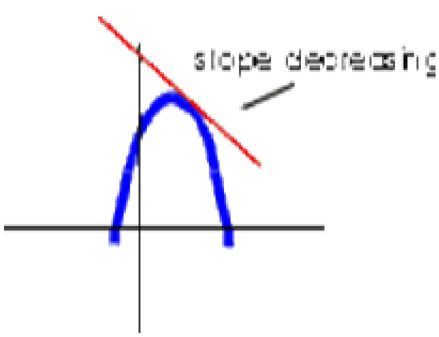
One of the convenient uses for the derivative of a function is finding the maximum or minimum points of the function.

The derivative indicates the rate of change of a function.
Put in other words, the derivative measures the slope of the function at a particular point.

When the slope changes from positive to negative, the function is at **its maximum** when the slope is zero.
When the slope changes from negative to positive, it is at **its minimum** when the slope is zero.







(We *will provide students with activities so that they can use the knowledge and the skills acquired in the Unit.*
They will develop these skills into higher skills . At the same time students will be given links with other subjects of the curriculum-)

Example:

APPLICATION 1 : Problems related to real life

George had a function of stresses on a particular part of airplane with respect to time in flight. He wanted to determine the time when the part was under the greatest stress.This function is $f(t) = -t^2 + 2t + 3$

Resolution

Given the function

$f(t) = -t^2 + 2t + 3$

George took the derivative of the function

$f'(t) = -2t + 2$

then he determined that the derivative was equal to zero at $t = 1$.

If we consider $-2t+2 > 0$ (greater than zero), we find that for $t < 1$, the first derivative is positive , and for $t > 1$ is negative.

Therefore, the time found of greatest stress was 1 minute after the take off.

APPLICATION 2 : Problems related to real life

Verify that the following function has stationary points at x= -1, 0, 1 :

$f(x) = 5\cdot x^6 + 12\cdot x^5 - 20\cdot x^3 - 15\cdot x^2 + 1$

$$f(x) = 5x^6 + 12x^5 - 20x^3 - 15x^2 + 1 \qquad \textbf{(28.1)}$$

Classify each of these three stationary points as a maximum, a minimum or a point of inflection.

Are there any other stationary points? If so, find them but do not attempt to classify them.

Resolution

First question

If we want to verify that the points x= -1, 0, 1 are stationary points, we have to calculate the first derivative of the function and then we have to verify that the derivative is equal to zero at these points:

$f'(x) = 30\cdot x^5 + 60\cdot x^4 - 60\cdot x^2 - 30\cdot x$

Let's substitute the value x=-1 :

$f'(-1) = 30\cdot(-1)^5 + 60\cdot(-1)^4 - 60\cdot(-1)^2 - 30\cdot(-1) = 0$
We can deduce that x=-1 is a stationary point.

Let's substitute the value x=0:
$f'(0) = 30\cdot(0)^5 + 60\cdot(0)^4 - 60\cdot(0)^2 - 30\cdot(0) = 0$
We can deduce that x=0 is a stationary point too.

Let's substitute the value x=1:
$f'(1) = 30\cdot(1)^5 + 60\cdot(1)^4 - 60\cdot(1)^2 - 30\cdot(1) = 0$
We can deduce that x=1 is a stationary point too.

Second question

First method
In order to classify these points we have to calculate the second derivative of the function and substitute these values:
$f'(x) = diff\left(5\cdot x^6 + 12\cdot x^5 - 20\cdot x^3 - 15\cdot x^2 + 1, x\right)$

$$\frac{d}{dx} f(x) = 30\,x^5 + 60\,x^4 - 60\,x^2 - 30\,x \quad \textbf{(28.1.1)}$$

$f''(x) = diff(30\,x^5 + 60\,x^4 - 60\,x^2 - 30\,x, x)$

$$\frac{d^2}{dx^2} f(x) = 150\,x^4 + 240\,x^3 - 120\,x - 30 \quad \textbf{(28.1.2)}$$

$f''(-1) = eval(150\,x^4 + 240\,x^3 - 120\,x - 30, x = -1)$

$$D^{(2)}(f)(-1) = 0 \quad \textbf{(28.1.3)}$$

This point could be a point of inflection. We have to verify it, so we have to calculate the third derivative and substitute the value of the point:
$f'''(x) = diff(150\,x^4 + 240\,x^3 - 120\,x - 30, x)$

$$\frac{d^3}{dx^3} f(x) = 600\,x^3 + 720\,x^2 - 120$$

$f'''(-1) = eval(600\,x^3 + 720\,x^2 - 120, x = -1)$

$$D^{(3)}(f)(-1) = 0 \quad \textbf{(28.1.5)}$$

$f''''(x) = diff(600\,x^3 + 720\,x^2 - 120, x)$

$$\frac{d^4}{dx^4} f(x) = 1800\,x^2 + 1440\,x \quad \textbf{(28.1.6)}$$

$f^{IV}(-1) = eval(1800\,x^2 + 1440\,x, x = -1)$

$$f(-1)^{IV} = 360 \quad \textbf{(28.1.7)}$$

$f(-1)^{IV} > 0$ therefore x=0 is a minimum point.
If we substitute the value in the function we can find the ordinate of the point

$f(-1) = eval\left(5 \cdot x^6 + 12 \cdot x^5 - 20 \cdot x^3 - 15 \cdot x^2 + 1, x = -1\right)$

$$f(-1) = -1 \qquad \textbf{(28.1.8)}$$

min(-1;-1)

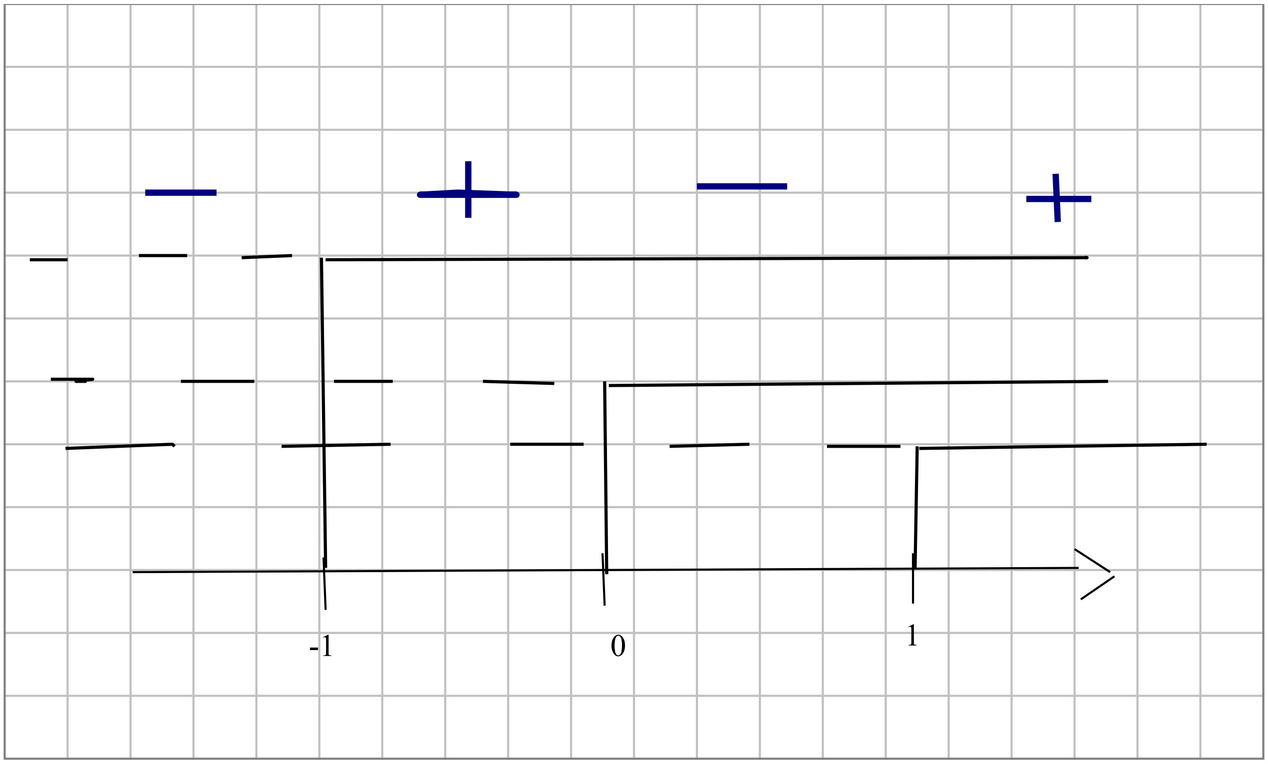
$f''(0) = 150 \cdot (0)^4 + 240 \cdot (0)^3 - 120 \cdot (0) - 30 = -30$
$f''(0) < 0$, therefore x=0 is a maximum point.
If we substitute the value in the function we can find the ordinate of the point
$f(0) = 5 \cdot (0)^6 + 12 \cdot (0)^5 - 20 \cdot (0)^3 - 15 \cdot (0)^2 + 1 = 1$
Max(0;1)

$f''(1) = 150 \cdot (1)^4 + 240 \cdot (1)^3 - 120 \cdot (1) - 30 = 240$
$f''(1) > 0$, therefore x=1 is a minimum point
Let's substitute the value in the function to find the ordinate of the point
$f(1) = 5 \cdot (1)^6 + 12 \cdot (1)^5 - 20 \cdot (1)^3 - 15 \cdot (1)^2 + 1 = -17$
min(1;-17)

Second method
If we want to classify these points we can also study te sign of the first derivative:
$30 \cdot x^5 + 60 \cdot x^4 - 60 \cdot x^2 - 30 \cdot x \geq 0$; if we consider the Ruffini's law we can factor in:

$x \cdot (x-1) \cdot (x+1)^3 \geq 0$

$$\begin{array}{lcl} x \geq 0 & & x \geq 0 \\ x - 1 \geq 0 & ; & x \geq 1 \\ (x+1)^3 \geq 0 & & x \geq -1 \end{array}$$



If we consider the point x=-1, the curve decreases on the left and increases on the right, therefore x=-1 is a point of minimum. Its coordinates are :

$$f(-1)=5\cdot(-1)^{6}+12\cdot(-1)^{5}-20\cdot(-1)^{3}-15\cdot(-1)^{2}+1=-1$$

Min(-1;-1)

If we consider the point x=0, the curve increases on the left and decreases on the right, therefore x=0 is a point of maximum. Its coordinates are :

$$f(0)=5\cdot(0)^{6}+12\cdot(0)^{5}-20\cdot(0)^{3}-15\cdot(0)^{2}+1=1$$

Max(0;1)

If we consider the point x=1, the curve decreases on the left and increases on the right, therefore x=1 is a point of minimum. Its coordinates are :

$$f(1)=5\cdot(1)^{6}+12\cdot(1)^{5}-20\cdot(1)^{3}-15\cdot(1)^{2}+1=-17$$

Min(1;-17)

Third question

If we want to find any other stationary points we have to consider the first derivative equal to zero and calculate its solutions:

$$30\cdot x^{5}+60\cdot x^{4}-60\cdot x^{2}-30\cdot x=0\ ;\qquad x\cdot(x-1)\cdot(x+1)^{3}=0\ ;\quad x=0\lor x=1\lor$$

$$(x+1)^{3}=0\quad ;\qquad x=0\lor x=1\lor x=-1$$

Since the solutions of the equation do not include other values, there aren't any other stationary points.

APPLICATION 3 : Problems related to real life

The path of a football is given by the equation $y = x - \frac{x^2}{40}, x \geq 0$.

If $\frac{dx}{dt} = 10\sqrt{2}$ for all t, find $\frac{dy}{dt}$ when $x = 10$.

DATA

$x'(t) = \frac{dx}{dt} = 10\sqrt{2}$

$x(t) = 10$

Resolution

We know that this equation can be considered a composite function because of the data that the problem provides us.

So, the function becomes $y = x(t) - \frac{[x(t)]^2}{40}$ and we can solve it with the derivation rules we have studied.

In fact, we have to calculate the first derivative of the function y and we obtain:

$$y' = x'(t) - \frac{2 \cdot x(t) \cdot x'(t)}{40} = x'(t) - \frac{x(t) \cdot x'(t)}{20}.$$

Replacing at the same time the value of first derivative x'(t)= $10\sqrt{2}$ and the value x(t)=10 in the function, we obtain:

$$y' = 10\sqrt{2} - 10 \cdot \frac{\sqrt{2}}{2} = 10\sqrt{2} - 5\sqrt{2} = 5\sqrt{2}$$

APPLICATION 4 : Problems related to real life

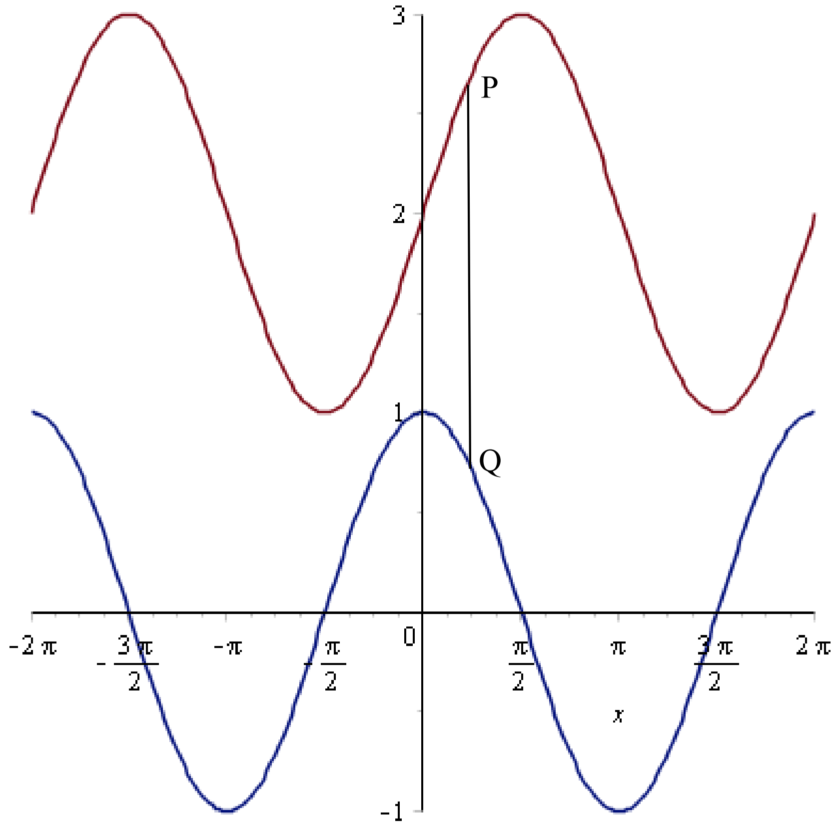
What is the minimum vertical distance between the graphs of $2 + \sin(x)$ and $\cos(x)$?

Let's consider the functions:

$f(x) := 2 + \sin(x)$ $g(x) := \cos(x)$

Resolution

The domain of these functions is R.

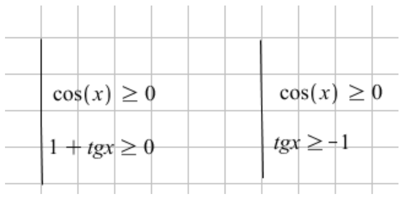


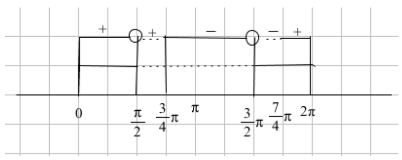
To calculate the minimum **vertical** distance between the two functions,we consider a point of the first function whose coordinates are P(x ; *2+sin(x)*) and a point of the second function whose coordinates are Q*(x; cos(x))*.
Since the distance is a vertical line, the abscissas of the two points are equal, so we calculate this distance as the absolute value of the difference between the ordinates:
$d = |2 + \sin(x) - \cos(x)|$: Since the function $\mathrm{y} = 2 + \sin(\mathrm{x})$ has all the values that belong to the range [1,3] and the function $\mathrm{g}(\mathrm{x}) = \cos(\mathrm{x})$ has all the values that belong to the range [-1,1], their difference will be positive or at least zero.
Therefore we can eliminate the absolute value:
$d = 2 + \sin(x) - \cos(x)$:Now we calculate the first derivative of the given function
$d' = \cos(x) + \sin(x)$: and we have to pose it greater or equal to zero
to study its sign and so to find its minimum and maximum points:

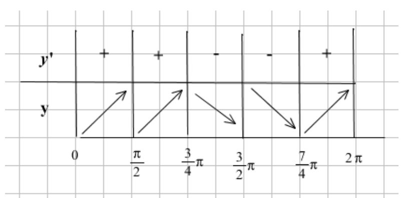
$\cos(x) + sen(x) \geq 0; \cos(x) \cdot (1 + tgx) \geq 0;$

Since *cos(x) is* a periodic function of period 2π, we'll consider the range [0; 2π] to

write the solutions of the inequality
Since *tg(x)* is a periodic function of period π, we'll consider the range [0; π] to write the solution of the inequality.

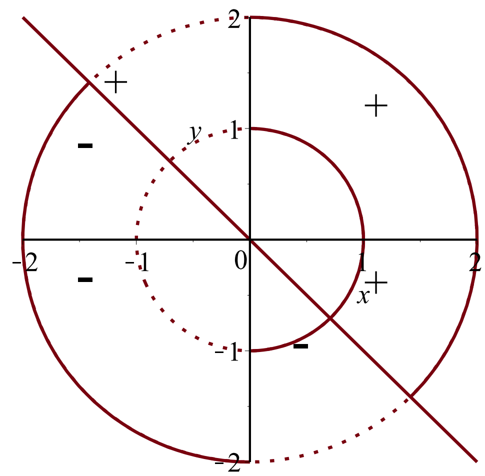






▶ **grafico**

$display(a, b, d, e, f, g, l, h, m, n, o, p)$



Since we want to know the minimum vertical distance, we'll consider the point $x = \frac{7}{4} \cdot \pi$ wich is the minimum of the funciton. If we substitute this value in the function we will find:

$d\left(\frac{7}{4}\pi\right) = 2 - \frac{(\sqrt{2})}{2} - \frac{(\sqrt{2})}{2} = 2 - \sqrt{2}$ that is the minimum vertical distance between the graph of
y=2+*sin(x)* and y=*cos(x)*

APPLICATION 5: Problems related to real life

A piece of wire, of length 20 cm, is to be cut into two parts. One of the parts, of length x cm, is to be formed into a circle and the other part into a square. Show that the sum, A cm^2, of the areas of the circle and the square that

A has a stationary when $x = \frac{20\,\pi}{4+\pi}$.

▶ **DATA**

▶ **Resolution**

▼ **APPLICATION 6: Problems related to real life**

The window of Sammy

Sammy the Owl wants to design a window that is a rectangle with a semicircle on top. If the total perimeter is constrained to be 24 feet, what dimensions should Sammy pick so that the window admits the greatest amount of light?
Give the radius of the semicircular region and the height of the rectangular portion.

▶ **Analysis of the situation**

▶ **Let's represent the graphic of the window**

▶ **Resolution**

▶ **Final resault**

SUMMARY of DERIVATIVES OF SOME COMMON FUNCTION

DERIVATIVES RULES

⩾ **Derivative of a constant** $D(k) = 0$	**The derivative of a constant is equal to zero.**
⩾**Derivative of x** $D(x) = 1$	**The derivative of the function x is equal to one.**
⩾**Derivative of x^n** $D(x^n) = n \cdot x^{n-1}$	**The derivative of the function x^n (x to the n) is equal to n multiplied by x to the n minus one.**
⩾ **Derivative of x^α** $D(x^\alpha) = \alpha \cdot x^{\alpha - 1}$	**The derivative of the function x^α (x to the alpha) is equal to alpha multiplied by x all alpha minus one.**
⩾**Derivative of a^x** $D(a^x) = a^x \cdot \ln a$ ⩾**Derivative of e^x** $D(e^x) = e^x$	**The derivative of exponential function a^x (a all x) is equal to a all x multiplied by natural logarithm of a.**
⩾**Derivative of $\log_a(x)$** $D(\log_a(x)) = \frac{1}{x} \cdot \frac{1}{\ln a}$	**The derivative of exponential function e^x (*e* all x) is equal to e all x** **The derivative of function $\log_a(x)$ (logarithm base *a* of x) is equal to inverse of x multiplied by inverse of natural logarithm of *a***
⩾**Derivative of $\ln(x)$** $D(\ln(x)) = \frac{1}{x}$	
⩾**Derivative of $\sin(x)$** $D(\sin(x)) = \cos(x)$	**The derivative of function $\ln(x)$(natural logarithm of x) is equal to one divided by x.**

⪰Derivative of $\cos(x)$ $D(\cos(x)) = -\sin(x)$	The derivative of function $\sin(x)$ (sine of x)is equal to cosine of x. The derivative of function $\cos(x)$ (cosine of x) is equal to minus of sine of x

By using the derivative rules in combination, we can find the derivatives of many other functions

Here are some basic laws which can be used to derive other differentiation rules.

Derivative of $k \cdot f(x)$ (the product of a constant and a function)

The derivative of the product of a constant and a function of x is equal to the constant multiplied by
the first derivative of the function.

$$D[k \cdot f(x)] = k \cdot f'(x)$$

Derivative of the sum of two functions

The derivative of the sum of two functions is equal to the sum of the first derivatives of the functions.

$$D[f(x) + g(x)] = f'(x) + g'(x)$$

Derivative of a product of two functions

The derivative of a product of two functions is equal to the product of the derivative of the first function multiplied by the second function, plus the first function multiplied by the first derivative of the second.

$$D[f(x) \cdot g(x)] = f'(x) \cdot g(x) + f(x) \cdot g'(x)$$

Derivative of the reciprocal function

The derivative of the reciprocal function is equal to the opposite of the first derivative of the function divided by the function all squared.

$$D\left[\frac{1}{f(x)}\right]=-\frac{f'(x)}{f^2(x)}$$

Derivative of the quotient of two differentiable functions

The derivative of a fraction, that is, the quotient of two differentiable functions, is equal to the function in the denominator multiplied by the derivative of the function in the numerator, minus the function in the numerator multiplied by derivative of the function in the denominator, all divided by the square of the function in the denominator.

$$D\left[\frac{f(x)}{g(x)}\right]=\frac{f'(x)\cdot g(x)-f(x)\cdot g'(x)}{g^2(x)}$$

Theorem of the derivative of a composite function y = f (g (x))

Given the composite function y = f (g (x)) (y equal to f to g x), the derivative of a function f of another function g of x is equal to the first derivative of f with respect to z multiplied by the derivative of g with respect to x.

$$D[f(g(x)]=f'(g(x))\cdot g'(x)$$

Theorem of the derivative of the inverse of a function

Given a function y = f (x) invertible and differentiable at an interval I, and let
x = F (y) be its inverse function.
At the points where f ' (x) ≠ 0 , the inverse function is differentiable and the derivative of the inverse of a function is equal to the reciprocal of the first derivative of the function given.

$$D[F(y)]=\frac{1}{f'(x)}$$

Derivatives of some inverse functions

Derivative of arcsin(x)	**The derivative of arc sine of x is equal to one divided by the**

$$D[\arcsin(x)] = \frac{1}{\sqrt{1 - x^2}}$$	**square root of the difference between one and the square of x**
Derivative of arccosin(x) $$D[arcos(x)] = -\frac{1}{\sqrt{1 - x^2}}$$	**The derivative of arc cosine of x is equal to minus one divided by the square root of the difference between one and the square of x**
Derivative of arctg(x) $$D[arctg(x)] = \frac{1}{1 + x^2}$$	**The derivative of the arctangent of x is equal to one divided by one plus the square of x**
Derivative of arccotg(x) $$D[arccotg(x)] = -\frac{1}{1 + x^2}$$	**The derivative of the inverse cotangent of x is equal to minus one divided by one plus the square of x**